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The economic and environmental performance of cogeneration under the Public Utility Regulatory Policies Act

by

Shantha Esther Daniel

A dissertation submitted to the graduate faculty

in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

Major: Industrial Engineering

Program of Study Committee: K Jo Min, Major Professor Douglas Gemmill Michael Larsen John Miranowski Lizhi Wang

Iowa State University

Ames, Iowa

2009

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ABSTRACT

In this dissertation, we formulate and analyze a series of electric utility-cogeneration facility relationships to understand their ramifications on the economic welfare and environment. For our models we focus on a host utility and a qualifying facility under Public Utilities Regulatory Policies Act (PURPA; 1978 and subsequent amendments) and the total surplus as the economic welfare performance criterion and the total nitrogen oxides (NO_x) emissions as the environmental performance criterion. We first model the host utility and qualifying facility interaction as a Stackelberg game and derive the equilibrium generation quantities, prices and total surplus without emission considerations. We show analytically that the total surplus when the host utility and qualifying facility interact due to PURPA is lower than when the cogeneration facility is an Independent Power Producer or IPP. The Independent Power Producer configuration is when the cogeneration facility sells electricity directly to retail electricity customers without a PURPA contract at the prevailing electricity price set by the electric utility. Next, we extend the basic model by considering the regulation of emissions of NO_x by the electric utility. The regulatory program is modeled after the Clean Air Interstate Rule's (CAIR; 2005 and subsequent amendments) ozone season NO_x program. By comparing the total NO_x emissions generated in the system in the cogeneration under PURPA or CGP configuration with the IPP configuration we show analytically that the total NO_x emissions is lower in the CGP if the heat demand of the thermal host attached to the qualifying facility is high and the PURPA buyback price at which the qualifying facility sells electricity to host utility is low. Through this study we have derived conditions under which PURPA is justified or and clarified the applicability of PURPA.



CHAPTER 1. INTRODUCTION AND BACKGROUND

1.1 Introduction

Energy is the foundation on which modern society has been built. Every aspect of energy – its generation, distribution and consumption – has become a topic of discussion and research in the last century. In particular, the efficient and reliable generation and supply of electricity has been important requirement to our everyday life. In the US, the electricity supply industry is heavily regulated to ensure that electricity is generated in an efficient and reliable manner with minimal damage to the environment. In this thesis we study a specific energy generation technology, cogeneration, and a specific legislation that aims to promote it, the Public Utility Regulatory Policies Act (PURPA). Specifically, we study the generation planning problem and the associated economic and environmental performance of a cogeneration facility and an electric utility that are part of a PURPA contract.

The objective of the thesis is to identify PURPA's impact on the generation and operation decisions of an electric utility and a cogeneration facility. In addition, it also aims to evaluate PURPA's provisions for the cogeneration facility by comparing the economic and environmental performance of the electric utility and cogeneration facility under PURPA with their performance in other configurations without PURPA.

To achieve the above mentioned objectives, we develop and study a series of progressively complex models that quantify and compare the total surplus of different energy generation/consumption system configuration. The components of the energy system are a vertically integrated electric utility, the retail electricity customers and a cogeneration facility with a thermal host with fixed heat and electricity demands. The main model of the paper is



one that models the system where the electric utility has a PURPA contract with a cogeneration facility. The model is referred to as the Cogeneration Generation Planning under PURPA or CGP model. To evaluate the economic and environmental performance of the CGP model, the CGP model's outputs are compared with three benchmark models. Each of the three benchmarks represents a different generation/ consumption strategies available to the components of the energy system.

The simplest benchmark is the Heat Production without cogeneration configuration or HP model. The HP model captures a system that does not any cogeneration in it. In this model a heat production unit owned by the thermal host satisfies the process heat demand of the thermal host. To satisfy its electricity demand the thermal host purchases electricity from the electric utility in a bilateral transaction

The second benchmark is an energy system configuration that consists of a cogeneration facility owned by the thermal host that satisfies both its heat and electricity demand but does not interact with the electric utility or the retail electricity customers. This model is referred to as the Self generation using cogeneration (SCG) model.

The third and most complex benchmark is an energy system in which a cogeneration facility and electric utility compete to supply electricity to the retail electricity customers. The cogeneration facility owned by the thermal host satisfies the heat and electricity demand. In addition, the cogeneration facility also supplies electricity directly to the retail electricity customers. This benchmark is based on the existence of Independent Power Producers (IPP) that serve retail load. The electric utility also supplies electricity to the retail electricity



customers. The cogeneration facility is referred to as an independent power producer and the model is called as cogeneration facility as an Independent Power Producer (IPP) model.

The thesis consists of two parts – the economic performance of cogeneration under PURPA and the environmental performance of cogeneration under PURPA. The economic performance of cogeneration under PURPA is evaluated by determining the total surplus realized in the CGP model with the total surplus realized in the benchmark models. The environmental implications of electricity generation well documented, it is important to evaluate the environmental performance of all regulations. Hence we evaluate the environmental performance of PURPA by comparing the total No_x emissions realized in the CGP models.

The rest of the thesis is organized as follow, in section 1.2 we present the background information on cogeneration and PURPA. In chapter 2 we focus on the economic performance of cogeneration under PURPA. Relevant literature related to the cogeneration under PURPA model is presented in section 2.1 followed by the modeling assumption of the basic models without environmental considerations in section 2.2. In section 2.3, 2.4, 2.5 and 2.6 we formulate and solve CGP model, IPP model, SCG model and the HP model respectively. In section 2.7 we compare the economic performance of the CGP model with the economic performance of the three benchmark models and summarize our findings regarding the economic performance of the cogeneration under PURPA configuration.

In chapter 3 we focus on the environmental performance of cogeneration under PURPA by formulating and solving the extended model which is the basic model from chapter 2 with environmental considerations. An overview of the NO_x regulatory programs is



presented in section 3.11 followed by a brief review of literature related to NO_x regulatory programs and cogeneration in section 3.2. In section 3.3 we present the additional modeling assumptions related to the extended model. In section 3.4, 3.5, 3.6 and 3.7 we formulate and solve CGPE model, IPPE model, SCGE model and the HPE model respectively. In section 3.8 we compare the total NO_x emissions of the CGPE model with the total surplus of the three benchmark models and summarize our findings regarding the environmental performance of the cogeneration under PURPA configuration. We conclude with chapter 4 with conclusions, discussion and future research.

1.2 Background

In this subsection the two critical components of the dissertation namely – cogeneration and PURPA are explained in detail.

1. 2.1 Overview of Cogeneration

Cogeneration is defined as the sequential use of fuel for generation of two useful energy products – electricity and useful heat (Petchers, 2003). Cogeneration has a fuel efficiency of 80 to 90 % compared to the 33% fuel efficiency of conventional electricity generation units. A conventional electric utility is one which generates only electricity and does not reutilize the waste heat energy in the exhaust gases or steam left over after electricity generation. A pictorial description of cogeneration and its variation from conventional energy production is presented in Figure 1.





Figure 1. Cogeneration process and its variation from conventional energy production¹

Cogeneration has two important advantages – fuel conservation and reduced emissions (Hu 1985; Petchers, 2003; Spiewak and Weiss, 1997). Cogeneration facilities are ideally suited for industries and facilities that require a reliable and continuous supply of both electricity and thermal energy. Examples of industries that are suitable for cogeneration include paper and pulp industries, oil refineries, chemical plants, etc. It is essential that the thermal and power loads of these industries should be closely matched. Cogeneration plants are also found in hospitals and universities. Cogeneration is preferred in such facilities since they require steady and reliable supply of power and heat for space heating.

Cogeneration facilities differ mainly in three ways - the type of fuel used, the technology and the load characteristics. In terms of fuel most cogeneration facilities operate

¹ Image courtesy of Cogenworks (http://www.cogenworks.com/gtf_CogProcess.html)



on Natural Gas or Oil. However, coal, biogas and other waste stream based fuels are also used by cogeneration facilities.

In terms of technology the main types of cogeneration facilities are topping cycle or bottoming cycle. In topping cycle cogeneration unit's electricity is generated first and the thermal output (heat or steam) is the secondary output. In the bottoming cycle cogeneration units fuel is used to generate the thermal output first and the waste heat is used to obtain electricity as the secondary product. In terms of the load characteristics or demand characteristics, cogeneration units are of two types –electricity load following and thermal load following. In the former type of cogeneration facility the electricity load satisfaction is the main goal with the operation of the unit dependent on the electricity demand. In the thermal load following type of cogeneration facility the thermal load satisfaction is the main goal with the operation of the unit dependent on the thermal load satisfaction is the main focus on cogeneration facilities that are topping cycle, thermal load following, cogeneration facilities

1.2.2 Public Utilities Regulatory Policies Act of 1978 (PURPA)

In the United States, cogeneration gained importance with the implementation of the Public Utility Regulatory Policies Act (PURPA) of 1978. The main intents of PURPA were

- To promote energy production efficiency and energy conservation
- Decrease dependence on foreign fuel sources and
- To promote use of renewable fuel sources



It achieved this purpose by promoting cogeneration (fuel efficiency) and small power producers (Danielsen et al, 1999, Lamoureux, 2002). To achieve its goals PURPA provided the status of "qualifying facility" to cogeneration and small power producers that satisfied certain efficiency standards. Qualifying facilities were provided with the following benefit

- 1. The right to sell electricity to the electric utility at the utility's "avoided cost".
- 2. Interconnection to the grid from the electric utility at non discriminatory rate and quality.
- 3. Availability of backup/maintenance/supplemental electricity at non discriminatory rates from the electric utility.

The utilities that bought the electricity sold by qualifying facilities and provided these facilities with interconnection to the grid were termed as the host utilities. The right to sell electricity to the electric utility at the utility's "avoided cost" based rate is termed as the "PURPA Put" (Gottlieb, 2001). The "avoided cost" based rate is defined as the cost that the utility would have incurred by generating or purchasing the electricity provided by the qualifying cogeneration facility (Hirsh, 1999, Danielsen et al, 1999). The law also ensured that the qualifying cogeneration facilities will be able to purchase backup/ maintenance/ supplemental at just and reasonable rates (Spiewak and Weiss, 1997). It should be noted that the rates charged by the electric utilities vary for backup electricity, maintenance electricity and supplemental electricity. The qualifying facilities were able to negotiate beneficial rates for backup and maintenance electricity. However, qualifying facilities are charged at the market price for supplemental electricity (Glassman, 2007)



Though PURPA is a federal law under the Federal Energy Regulatory Commission, the implementation of the law is by state regulatory authorities. As part of the implementation of PURPA, electric utilities entered into long term contracts with qualifying facilities. These contracts were executed in the early eighties to nineties for periods ranging from 10 to 30 years. For example, in California the PG&E utility entered into a contract with Watson Cogeneration Facility in 1985 which will expire in 2008 (Hawley, 2005). Warrior Run a 180 MW facility in Maryland has a PURPA contract that will expire in 2029 (Wilson et al, 2005). In addition to overseeing the establishment of these long term contracts, the state authorities are also responsible for determining the "avoided cost" of the utilities (Hirsh, 1999). The definition of avoided cost has been interpreted in many ways by the various state authorities and different formulas for its calculation are in effect in the different states (Spiewak and Weiss 1997, pg 29).

The effect of PURPA and the long term PURPA contracts has been pronounced with both supporters and detractors for the regulation. The most contentious aspect of the law is the PURPA put and the estimation of avoided costs. The point of contention between the utilities and the qualifying facilities has been the above market rate of the avoided cost (Danielsen et al, 1999). Utilities claim that the high avoided costs and the mandatory purchase obligation are directly responsible for high wholesale prices (EIA, 2000). Detractors claimed that nearly \$42 billion will be paid by consumers of electricity in above market prices for PURPA mandated electricity from qualifying facilities and that PURPA



utilities claim that the law is anticompetitive and does not allow for flexibility in utility generation planning (Lamoureux, 2002)

Qualifying facilities claim that the PURPA incentives are crucial for their viability as electricity generators and that they do not have access to competitive markets. PURPA supporters state that the deregulation has not been completely successful and that the law is required for continued sustainability of the qualifying facilities (Lamoureux, 2002). Supporters also claim that the intents of the law, energy conservation and fuel diversification, are still critical to the nation and that the current industry practices do not promote these goals. (EIA,2000).

In 2005, the most comprehensive Energy Bill, the Energy Policy Act (EPAct) was passed by the congress. Based on the concerns raised by the utilities, EPAct amended PURPA in the following critical aspects

- Electric utilities are not required to enter into new contracts with qualifying facilities to purchase or sell electrical energy when qualifying facilities have nondiscriminatory access to wholesale competitive markets.
- 2. Qualifying facilities can be owned by electric utilities
- 3. Qualifying cogeneration facilities have to demonstrate that their thermal/heat output is useful.



However, the relief from mandatory purchase and mandatory sell obligations are not granted automatically to the existing contracts (PURPA, Title 18, § 292.309). Utilities are required to make an appeal to the FERC and decisions will be made on a case by case basis.

The total installed qualifying facility, as of 2000, was 45, 813 MW (Electric Power Annual, 2000). As of 2005, combined heat and power or cogeneration capacity in the United States is 66.9 thousand MW's. (EIA-Electricity Capacity, 2006). This capacity is only 6.8% of the total generation capacity available in the country. Coal was the major fuel source for electricity generation. Nearly 49.7% of electricity in 2005 was generated using coal. Natural gas (18.7%) and Petroleum (3%) were the next mostly used fuel source. Renewable and nonconventional fuel sources contributed only to about 2.7% of the total electrical energy generation. (EIA-Electricity Generation, 2006). These statistics give credence to the claims of the qualifying facilities and PURPA contract supporters. Given these conflicting claims and with the changes in the electricity industry due to re-structuring, it is crucial to evaluate the performance of cogeneration under PURPA and identify conditions under which it is justified.



CHAPTER 2. ECONOMIC PERFORMANCE OF COGENERATION UNDER PURPA

In chapter 2 of the thesis we study the economic performance of cogeneration under PURPA. The chapter begins with a review of relevant literature followed by the mathematical modeling and solution of the four models – the CGP model and the three benchmark models. We then compare the total surplus associated with the optimal generation plans of the electric utility and cogeneration facility in each configuration to study the economic performance of cogeneration under PURPA.

2.1 Literature Review

Various aspects of cogeneration have been studied as early as 1970's. The implementation of PURPA in 1978 brought a host of studies on cogeneration and PURPA in particular. Studies such as, Puttgen and MacGregor (1989), Moleshi et al (1991), Venkatesh and Chankong (1995) and Chen and Hong (1996), focused on the optimal production policy for a cogeneration facility. The focus of these studies was more on the interpretation of the physical constraints of a cogeneration unit and determining the production policy that guarantees minimum cost. Algorithms with emphasis on economic dispatching of cogeneration units also came into focus and are still of interest as demonstrated by Guo et al (1996), Rao (2001) and Chapa and Galaz (2004). While Guo et al (1996), Rao (2001) and Chapa and Galaz (2004). While Guo et al (1996), Rao (2001) and Chapa and Galaz (2004) was the only one to explicitly consider the feasible region constraints associated with cogeneration.



Joskow and Jones (1983) are one of the earliest studies on the economics of cogeneration. They focused on identifying conditions which lead to a profitable investment in cogeneration and concluded that the PURPA buyback rate, electricity prices and variable cost of operation determine the profitability of a cogeneration investment. Rose and McDonald (1991) also studied the economics of self-generation but concluded that the PURPA buyback rate by itself was not a significant determinant in the generation planning of cogeneration facilities. They concluded based on empirical evaluation that the electricity and steam demand faced by the cogeneration facility determine the generation plans of the facility. Woo (1988) studied the inefficiency of avoided cost pricing in a regulated environment where the regulator is interested in maximizing social welfare. The paper concluded that in regulated systems with a positive profit criteria the avoided cost pricing mentioned in PURPA will be in efficient with respect to social welfare.

Fox-Penner (1990) is one of the papers that focused on the strategic operational decisions of a cogeneration facility under PURPA. Cogeneration facilities have two incentives to generate electricity – to minimize cost of purchasing electricity from electric utilities to satisfy their demand (displacement mode) and to profit from selling electricity to the electric utilities or market customers (arbitrage mode). Fox – Penner (1990) explicitly models the two modes with respect to a PURPA cogeneration facility. However, he does not consider the impact of the cogeneration facility's generation decisions on the electric utility or strategic gaming between the electric (host) utility and the cogeneration (qualifying) facility.



Haurie et al (1992) study the strategic gaming between the host utility and the qualifying facility. They model the interaction as a Stackelberg game with the host utility as the leader and the qualifying facility as the follower. Though Haurie et al's (1992) is similar to our model, they differ in certain critical and salient points. Haurie et al (1992) do not explicitly consider the PURPA constraint that specifies that the qualifying facility can only sell electricity that it has cogenerated to the host utility and the fixed nature of the PURPA buyback price that the host utility pays to the qualifying facility. In their model this price is endogenously determined. This is in direct conflict with concerns raised by the conventional utilities regarding the PURPA contracts (Hirsch, 1999). If electric utilities had the freedom to choose the price at which they purchase electricity from the cogeneration facility then they would set the price in a manner that is beneficial for them and will be more amenable to the PURPA contracts. However, this is not the case and there is evidence that electric utilities are even willing to buy out PURPA contracts so that they need not accept cogenerated electricity (Danielsen, 1999). In addition, Haurie et al (1992) assume that the conventional utilities sell electricity at their average cost and buy electricity from the cogeneration facility at the utility's marginal cost. Due to deregulation, utilities no longer find it profitable to sell electricity at their average cost and are currently selling at their marginal cost to be competitive. In addition, if the cogeneration facility was selling to the electric utility at the host utility's marginal cost, then the host utility will be indifferent to the quantity of electricity sold to them by the cogeneration facility. Anecdotal evidence suggests otherwise (Hirsh 1999, Danielsen 1999).



Kwun and Baughman (1991) is another important paper that focused on the benefits of cogeneration and the optimal investment in cogeneration capacity. To determine the benefits of cogeneration, the study determines the optimum generation decisions and associated cost of an electric utility and an industrial facility that faces thermal demand without cogeneration individually. They then compare the cost with a jointly optimized global benchmark model with cogeneration.

This thesis differs from the above studies in two ways. The studies in literature that evaluate the strategic interaction of the cogeneration facility and the electric utility under PURPA were before deregulation came into effect. The second aspect not considered in available literature on cogeneration is the economic welfare implications of cogeneration under PURPA.

2.2 Modeling Assumptions

A1:The heat demand S_d of the thermal host is much greater than the thermal host's electricity demand Q_d . Examples of such a cogeneration facility that serve such cogeneration/thermal host include a 20 MW Pacific Cogeneration facility in Washington. The facility provides electricity and heat to the Greater Western Malting facility with an average electrical load of 4MW and a heat load of 90 MBtu/hr ("PURPA Resource Development" 1990).

A2: The relationship between cogenerated electricity and heat is assumed to be a fixed ratio (power to heat ratio).

A3: The cogeneration facility can discard excess heat or electricity without any disposal cost.A4: The electric utility is a vertically integrated utility. Vertically integrated utilities are



companies that handle all three aspects of electricity supply-generation, transmission and distribution. The customers of vertically integrated utilities are the retail customers comprising of the residential sector (residential homes), commercial sector (office and store buildings) and industrial sector. Vertically integrated utilities are still a part of the United States electricity supply sector. Even though de-regulation has been in effect since late 1990's, a significant part of the country is still served by regulated monopolistic vertically integrated utilities. Even in states with a wholesale competition, parts of the state might be served by vertically integrated utilities due to transmission network constraints. For example in Texas, 15 % of the state's load is outside of the ERCOT grid and include regions in the Texas pan handle, parts of northeast Texas, southeast Texas and El Paso and surrounding areas (AECT, 2007). These regions are serviced by vertically integrated utilities such as Southwestern Public service (pan handle), the Southwestern Electric Power Company (Northeast Texas), Entergy (Southeast Texas) and The Electric Company (El Paso and surrounding areas). Another example of a region served by a monopolistic electricity generator includes San Francisco (Borenstein and Bushnell, 2000)

A5: In the CGP, HP and SCG models, the electric utility is the sole supplier of electricity to the retail electricity customers. In the IPP model the electric utility and cogeneration facility supply electricity to the retail electricity customers. However, the market structure is similar to a monopoly with a fringe with the electric utility as the monopoly and the cogeneration facility as the fringe supplier. With deregulation the United States has seen the emergence of Non-Utility generators or NUG's. These NUG's do not own transmission or distribution services and hence use the utility's transmission and distribution services to deliver



electricity to wholesale electricity customers or retail electricity customers(Philipson and Willis, 2006). NUG's are mainly three types – Independent Power Producers, Qualifying facilities and Exempt Wholesale Generators. Independent Power Producers are private companies that generate and consume electricity and sell surplus electricity. While wholesale competition is prevalent in many states across the country only a few states have implemented retail competition. However, in a few cases, NUG's participate in retail electricity sales. In 2006, data collected by the Energy Information Administration (EIA) using form EIA-920 had 116 NUG's that supplied electricity to retail customer. In each case the NUG's handled a small portion of the retail sales while the remaining was satisfied by the local utility (EIA, 2006a)

A6: The electric utility will always sell the cogenerated electricity it purchases from the cogeneration facility to its retail electricity customers.

A7: The models do not consider capacity constraints, transmission constraints or emission control constraints with regards to both the electric utility as well as the cogeneration facility. It is assumed that the electric utility being a vertically integrated utility has sufficient capacity to meet the maximum demand of the retail electricity customers and has access to transmission networks.

A8: When the electric utility and cogeneration facility are engaged in a Stackleberg game (CGP, CGS and IPP models), complete information is assumed between the two players.

In this thesis, for simplification purpose we use Mega British Thermal Units (MBtu) to specify all energy products. This is to specify both the electrical and heat energy output in



the same units, instead of Megawatt-hour for electricity (MWh) and MBtu for heat energy. We use the conversion 1 MWh = 3.4121 MBtu to convert electricity prices are generally specified in \$/MWh to \$/MBtu. An MBtu is equal to a million Btu's. In the paper, the variables and parameters are introduced with the appropriate units. After the initial definition, the variables and parameters are referred to without their units. For easy reference a list of all variables and parameters used in chapter 2 along with their definition and units is presented in Table 1

Symbol	Description		
Decision Variables:			
x_u	Electricity generated by electric (host) utility (MBtu).		
x_{cg}	Electricity cogenerated by the cogeneration (qualifying) facility (MBtu).		
y _{cg}	Heat energy cogenerated by the cogeneration (qualifying) facility (MBtu)		
q_s	Electricity sold by the qualifying facility to the host utility as part of the PURPA contract (MBtu).		
	(In CGP model only)		
q_b	Electricity purchased by the qualifying facility from the host utility as part of the PURPA contract		
	(MBtu). (In CGP model only)		
P_r	Electricity price paid by the retail electricity customers (\$/MBtu).		
Parameters:			
Q_d	Electricity demand of the service facility (MBtu).		
S _d	Heat demand of the service facility (MBtu).		
P_s	PURPA buyback price at which the qualifying facility sells electricity to the host utility as part of		
	the PURPA contract (\$/MBtu).		
α	Power to heat ratio of the cogeneration facility (constant).		

Table1. Notation in chapter 2



2.3 Cogeneration under PURPA (CGP) configuration model

In the CGP model the electric utility and cogeneration facility have an existing PURPA. The cogeneration facility is owned by the thermal host and together they are referred to as the qualifying facility. Since the cogeneration facility has a valid PURPA contract with the electric utility, it is referred to as the PURPA host utility. The qualifying facility sells electricity to the host utility at a pre-determined price stipulated in the PURPA contract. In the CGP model, as part of the PURPA contract, the qualifying facility can purchase electricity from the host utility at the prevailing electricity price in an independent transaction. The energy generation system configuration of the CGP model is shown below in Figure 2.



Figure 2. Cogeneration under PURPA (CGP) configuration model



In the CGP configuration the host utility and qualifying facility are engaged in a Stackleberg game with the host utility as the leader and the qualifying facility as the follower. The leader in a Stackelberg game is the dominant player who can actively make decisions that affect both players. In the CGP configuration, the host utility decides the electricity price to the retail electricity customers and is the dominant player and the Stackelberg leader. The follower is the less dominant player whose decisions are based on the decisions of the leader. In the CGP configuration, the qualifying facility takes the electricity price, P_r (\$/MBtu), as given and determines its optimal generation plan and is the Stackelberg follower.

The Stackelberg game is a sequential game with the leader making her decisions first and the follower making his decisions after observing the leader's decisions. (Gibbons, 1992, pg 58). In addition, the leader is aware of the fact that the follower will observe the leader's decision and then make his decision. It is assumed that both players have perfect information, i.e. both players are fully aware of their respective payoffs for any combination of strategy/action available to them. The Stackelberg game between the host utility and the qualifying facility is played as follows,

1. The host utility, as the leader, announces the electricity price, P_r . The host utility determines the electricity price by choosing its generation quantity, x_u (MBtu). However, the host utility as the leader, anticipates the response of the qualifying facility (follower) to the electricity price, P_r before he announces the price P_r . The generation quantity x_u is the action set of the leader (host utility). The payoff associated with the action set, is the profit that the host utility gains by selling electricity in the wholesale market at the



associated electricity price, P_r . It should be noted that the electricity sold by the host utility in the market includes the cogenerated electricity sold to it by the qualifying facility as part of the PURPA contract and excludes the electricity sold by the host utility to the qualifying facility.

- 2. The qualifying facility observes the electricity price P_r and assuming that its decisions will not have an impact on the electricity price makes the following decisions,
 - The quantity of electricity and heat to cogenerate, x_{cg} (MBtu) and y_{cg} (MBtu) respectively, given that the electricity demand, Q_d (MBtu), and heat demand, S_d (MBtu), of the thermal host has to be satisfied.
 - The quantity of electricity to purchase from the host utility q_b (MBtu) at the electricity price, P_r to satisfy part or all of the thermal host's electricity demand Q_d .
 - The quantity of cogenerated electricity to sell to the host utility q_s (MBtu) as part of the PURPA contract at the PURPA buyback price, P_s (\$/MBtu)

The factors that affect the qualifying facilities decisions are the cost of purchasing electricity from the host utility i.e. the electricity price (P_r) , the cost of cogeneration, $C_{cg}(x_{cg}, y_{cg})$ (\$), the electricity (Q_d) and heat demand (S_d) respectively of the thermal host and PURPA buyback price P_s , at which the qualifying facility can sell electricity to the host utility. Hence the action set of the qualifying facility is characterized by the set of decisions $(x_{cg}, y_{cg}, q_s, q_b)$ that satisfy certain constraints that will be described in section 2.3.1 of this chapter. The payoff associated with the decisions of the qualifying facility is the profit it gains by selling cogenerated electricity to the host utility at the PURPA buyback price P_s and the satisfaction of the energy demands of the thermal host.



The solution to the Stackelberg game described above is determined using backward induction (Gibbons, 1992, pg 57). Let x_l^* and x_f^* be the optimal strategies for the leader and the follower. In backward induction method, the follower maximizes his profit for each strategy of the leader i.e. determines $x_f^*(x_l)$. The function $x_f^*(x_l)$ is called the best response or reaction function of the follower. We then determine the leader's strategy that maximizes her profit, given the reaction/response of the follower, i.e. the leader identifies x_l^* that maximize his profit, $\pi_l(x_l, x_f^*(x_l))$. (Cachon et al (2003), Gibbons (1992))

In the Stackelberg game of the CGP model, we first determine the optimal generation plan of the qualifying facility for each value of the electricity price P_r . The generation planning problem of the qualifying facility is an optimization problem. The optimal solution to the qualifying facility's generation planning problem with the electricity price P_r as given is called the best response function or reaction function of the follower. We then find the host utility's generation plan that maximizes his profit, given the best response of the qualifying facility to his decisions.

The qualifying facility's generation planning problem is described in subsection 2.3.1 along with the reaction function of the qualifying facility. This is followed by the host utility's generation planning problem in subsection 2.3.2. The equilibrium solution to the Stackelberg game in the CGP configuration and the associated total surplus are described in subsection 2.3.3

2.3.1Qualifying facility's generation planning problem

The qualifying facility's generation planning problem is a profit maximization problem. The qualifying facility determines its optimal cogeneration outputs – electricity (x_{cg}) and useful



thermal energy or heat (y_{cg}) . The qualifying facility profits by selling electricity to the host utility and has to satisfy the electricity and heat demand of the thermal host.

The profit maximization problem (P1) of the qualifying facility is presented below.

$$\frac{Max}{q_s, x_{cg}, y_{cg}, q_b} \pi_{qf}^{cgp} = P_s q_s - C_{cg} (x_{cg}, y_{cg}) - P_r q_b$$
(1)

s.t
$$q_s \le x_{cg}$$
 (2)

$$y_{cg} \ge S_d \tag{3}$$

$$x_{cg} = \alpha y_{cg} \tag{4}$$

$$x_{cg} - q_s + q_b = Q_d \tag{5}$$

$$q_s \ge 0; q_b \ge 0 \tag{6}$$

where $C_{cg} = a + bx_{cg} + cx_{cg}^2 + dy_{cg} + ey_{cg}^2 + fx_{cg}y_{cg}$ is the cost of cogeneration. The cost of cogeneration is a quadratic function that is used frequently in the economic dispatch literature for cogeneration systems (Guo et al 1996; Rao 2001; Chapa and Galaz 2004). The profit function π_{qf}^{cgp} , of the qualifying facility consists of three terms corresponding to the revenue from selling cogenerated electricity to the host utility, the cost of cogeneration and the cost of purchasing electricity from the host utility.

The constraints in the qualifying facility's generation planning problem P1 can be classified as regulatory constraint(s), demand constraints, operational constraint(s) and non-negativity constraints. The regulatory constraint specified by inequality (1) regulates the simultaneous sale and purchase of electricity by the qualifying facility from the host utility (Fox-Penner, 1990; Spiewak and Weiss (pg 30), 1997). The constraint is part of PURPA to prevent qualifying facilities from exploiting host utilities by buying electricity from them (at



a lower price) and selling the same electricity back (to them at a higher price). The demand constraints associated with the electricity and heat demand of the thermal host are specified by equation (3) & (4) respectively. It is noted that the electricity demand constraint is a strict equality since any excess electricity is sold to the host utility. The heat demand is a lower bound on the heat output of cogeneration since it is possible that the qualifying facility might generate more heat than required to sell the associated additional electricity to the host utility. Equation (5) is the operational constraint that specifies the electricity cogenerated for each unit of useful heat that is generated by the cogeneration unit. The power-to heat ratio of the cogeneration unit is utilized to specify this relationship (Sundberg and Henning, 2002). The power to heat ratio is defined as the ratio of electricity/power production to useful heat/steam production of a cogeneration unit (Spiewak and Weiss, 1997). The non-negativity constraints in P1 are the two non-negativity constraints associated with the electricity trading variables, q_s and q_b specified by equation (6).

The decision variables y_{cg} and q_b are eliminated from problem P1 using the equality constraints (4) and (5). The four variable problem P1 is represented by the equivalent two variable profit maximization problem (P2) given below.

$$\begin{array}{l} Max \\ q_s, x_{cg} \end{array} \pi_{qf}^{cgp} = (P_s - P_r)q_s - \bar{C}_{cg}(x_{cg}) + P_r x_{cg} - P_r Q_d \tag{7}$$

$$s.t\frac{1}{\alpha}x_{cg} \ge S_d \tag{8}$$

$$q_s \le x_{cg} \tag{9}$$

$$q_s \ge x_{cg} - Q_d \tag{10}$$

$$q_s \ge 0 \tag{11}$$



where $\bar{C}_{cg}(x_{cg}) = a + c_1 x_{cg} + c_2 x_{cg}^2$ is the modified cogeneration cost with $\frac{c\alpha^2 + f\alpha + e}{\alpha^2} = c_2$ and $b + \frac{d}{\alpha} = c_1$.

The objective function π_{qf}^{cgp} of P2 is linear in q_s and quadratic in x_{cg} . Due to inequality (9) and by the assumption that S_d is much greater than Q_d , the non-negativity constraint on q_s becomes redundant. Hence equations (8) and (10) provide the lower and upper bound on q_s respectively. The objective function being linear in q_s , the optimal value of q_s will be at one of its bounds - the upper bound, x_{cg} or the lower bound, $x_{cg} - Q_d$. The criteria based on which the optimal value of q_s will be at its upper or lower bound is the variable's coefficient in the objective function. If $P_s > P_r$, the coefficient of q_s in π_{qf}^{cgp} is positive and hence the optimal value of q_s will be at its upper bound of x_{cg} . If $P_s < P_r$, the coefficient of q_s in π_{qf}^{cgp} is negative and hence the optimal value of q_s will be at its lower bound of $x_{cg} - Q_d$. If $P_s = P_r$, then the qualifying facility and the host utility are indifferent to the quantities of electricity traded between them.

The optimal value of q_s provides certain insight into the qualifying facility's strategy for trading electricity i.e. the qualifying facility either sells all the cogenerated electricity to the host utility or it satisfy's the thermal host's electricity demand and sells the surplus cogenerated electricity to the host utility. We include the reaction of the qualifying facility when $P_s = P_r$ with the reaction of the qualifying facility when $P_s < P_r$.

The qualifying facility's generation planning problem (P2) further reduces to the determining the optimal cogenerated electricity x_{cg} under two cases – Case (a) when $P_s > P_r$ and Case (b) when $P_s < P_r$.



Case (a): In this case, the objective of problem (P2) reduces further to give us problem (P3) as follows,

$$\frac{Max}{x_{cg}}\pi_{qf}^{cgp} = P_s x_{cg} - \bar{C}_{cg}(x_{cg}) - P_r Q_d \quad \text{s.t} \quad x_{cg} \ge \alpha S_d$$
(12)

The only constraint associated with this problem is the heat demand constraint. Problem P3 has a nonlinear objective and a linear constraint, the solution to which can be obtained from its first order necessary conditions (FONC). The first order necessary are also the sufficient condition since the profit function π_{qf}^{cgp} , is concave w.r.t x_{cg} are

$$\frac{\partial \pi_{qf}^{cgp}}{\partial x_{cg}} = P_s - c_1 - 2c_2 x_{cg} = 0 \tag{13}$$

$$x_{cg}^{*NB} = \frac{P_s - c_1}{2c_2} \tag{14}$$

where x_{cg}^{*NB} is the optimal non-binding solution to problem P3. When the constraint is active, the optimal binding solution is $x_{cg}^{*B} = \alpha S_d$. Therefore, we have that the optimal x_{cg} will be

$$x_{cg}^{cgp} = Max\{\frac{P_s - c_1}{2c_2}, \alpha S_d\}$$
(15)

Case (b): In this case (b), the objective of problem P2 is reduced to form problem (P4) as follows,

$$\frac{Max}{x_{cg}}\pi_{qf}^{cgp} = P_s x_{cg} - \bar{C}_{cg}(x_{cg}) - P_s Q_d \text{ s.t } x_{cg} \ge \alpha S_d$$
(16)

From equations (12) and (16) we see that the objective function in Case (a) and Case (b) differ only in the third constant term which can be ignored for the purpose of determining the optimal x_{cg} . It is interesting to note that the difference in the constant term of the objective functions in equation (12) and (16) is related to how the electricity demand Q_d , of the thermal host is satisfied. In equation (12) the term, P_rQ_d , is the cost to the qualifying facility in



purchasing electricity from the host utility to satisfy Q_d . In equation (14) the term, P_sQ_d , is the revenue forgone by the qualifying facility by satisfying the electricity demand of the thermal host from the cogenerated electricity x_{cg} instead of selling the same quantity to the host utility. From equation (14) the optimal heat cogenerated y_{cg} , is determined using equation (5) as

$$y_{cg}^{cgp} = Max\{\frac{P_{s} - c_{1}}{2c_{2}}, \alpha S_{d}\}$$
(17)

We note from equations (14) and (17) that optimal cogenerated electricity and heat are independent of P_r . However, the decision on how best to satisfy the electricity demand of the thermal host is dependent on the electricity price P_r . In effect, the qualifying facility is faced with two distinctive and separate optimization problems,

- 1. What are the optimal cogeneration outputs for a given set of cost, revenue and demand parameters?
- 2. How should electricity be traded with the host utility for a given set of cogenerated electricity and electricity price?

Reaction function of the qualifying facility: In the CGP model the qualifying facility and the host utility are engaged in a Stackelberg game. The qualifying facility being the follower reacts to the decisions of the host utility, the leader. The qualifying facility reacts to the electricity price P_r , by deciding the quantity of electricity that it wishes to trade with the host utility. As determined above, only the quantity of power traded depends on the electricity price. The cogeneration outputs of electricity and heat are independent of the electricity price. Therefore the reaction function of the qualifying facility to the electricity price P_r is given below



$$q_s^{cg} = \begin{cases} x_{cg} \text{ when } P_s > P_r \\ x_{cg} - Q_d \text{ when } P_s \le P_r \end{cases} \text{ and } q_b^{cg} = \begin{cases} Q_d \text{ when } P_s > P_r \\ 0 \text{ when } P_s \le P_r \end{cases}$$
(18)

Similar to Fox-Penner (1990) we refer to case (a) and case (b) as the arbitrage mode (AM) and non-arbitrage mode (NAM). The trading strategy of the qualifying facility is the basis for the names of the cases. In the arbitrage mode (AM), the qualifying facility sells the entire quantity of cogenerated electricity to the host utility and purchases electricity from the host utility to satisfy the thermal host's electricity demand. In the non-arbitrage mode (NAM), the qualifying facility satisfies the electricity demand of the thermal host from the cogenerated electricity and sells only the excess electricity to the host utility. In each mode of trading, the cogeneration facility will generate a quantity of electricity such that the associated heat is equal to the heat demand of the thermal host or a quantity of electricity such that the associated heat will be greater than the heat demand of the thermal host.

2.3.2 Host utility's generation planning problem

The generation planning problem of the host utility is also a profit maximization problem. The profit maximization problem (P5) of the host utility in the CGP configuration is as follows

$$\frac{Max}{x_u} \pi_{hu}^{cgp} = P_r(Q_T)Q_T - C_u(x_u) - P_s q_s(P_r) + P_r(Q_T)q_b(P_r)$$
(19)

where $P_r(Q_T) = \beta - \gamma Q_T$ is the inverse demand function of retail electricity customers and $C_u(x_u) = l + mx_u + nx_u^2$ is the quadratic cost function associated with electricity generation for the power level of x_u (Wood and Wollenberg, 1996). Q_T (MBtu) is the total electricity supply to the retail electricity customers. Equation (19) is the profit function of the host utility.


The total electricity supply to the retail electricity customers is a function of the host utility's generation quantity (x_u) , the cogenerated electricity sold to the host utility by the qualifying facility (q_s) and the electricity purchased by the qualifying facility from the host utility (q_b) .

$$Q_T = x_u + q_s(P_w) - q_b(P_w)$$
(20)

Using the equation (4) we can eliminate q_b from equation (20) to obtain

$$Q_T = x_u + x_{cg} - Q_d \tag{21}$$

Equation (21) states that the total electricity supply to the retail electricity customers is a function of the host utility's generation quantity (x_u) , the qualifying facility's cogenerated electricity quantity (x_{cg}) and electricity demand of the thermal host (Q_d) . The host utility being a Stackelberg leader is fully aware of this and makes her decision accordingly. It should also be noted that, since x_{cg} and Q_d are both remain constant, for all feasible values of x_u , the host utility has the freedom to choose an electricity price that maximizes its own profit. The profit function π_{hu}^{cgp} , of the host utility consists of four terms – revenue from electricity sales to the retail electricity customers, cost of generation, cost of purchasing cogenerated electricity from the qualifying facility as part of the PURPA contract and the revenue from selling electricity to the qualifying facility at the electricity price P_r in a bilateral transaction. The profit of the host utility is a function of the reactions of the qualifying facility.

Equation (18) gives the reaction function of qualifying facility w.r.t. the electricity price, P_r . To determine the Stackelberg equilibrium solution we substitute the value of q_s and q_b in problem (P4) and solve for the optimal x_u . From equation (18) we have the reaction



function of the qualifying facility to be a discontinuous function with the discontinuity occurring when the electricity price, P_r becomes greater than or equal to the PURPA buyback price, P_s , at which the qualifying facility sells power to the host utility. Since the electricity price is a function of the host utility's generation quantity the discontinuity of the reactions function at $P_r \ge P_s$ leads to the following condition on the generation quantity of the host utility, x_u

$$\frac{1}{\gamma}(\beta - P_s - \gamma(x_{cg} - Q_d)) \ge x_u \tag{22}$$

The reaction of the qualifying facility differs based on the generation quantity of the host utility and the electricity price P_r . The quantity of electricity traded by the qualifying facility remains a constant for all $P_r < P_s$. Similarly, the quantity of electricity traded by the qualifying facility remains a constant (different from when $P_r < P_s$) for all $P_r \ge P_s$. The two trading modes of the qualifying facility lead to two profit maximization problems for the host facility.

Non-Arbitrage Mode ($P_r \ge P_s$): In this case the reaction of the qualifying facility from equation (18) is substituted in the profit function π_{hu}^{cgp} of the host utility. The profit maximization problem (P6) of the host utility is modified as follows,

$$\sum_{x_{u}}^{Max} \pi_{hu}^{cgp} = \beta Q_{T} - \gamma Q_{T}^{2} - C_{u}(x_{u}) - P_{s}(x_{cg} - Q_{d})$$
(23)

s.t
$$x_u \leq \frac{1}{\gamma} (\beta - P_s - \gamma (x_{cg} - Q_d))$$
 (24)

The problem P6 of the host utility is a simple non-linear problem. The corresponding Lagrangian function is

$$L_{hu}^{cgp} = \beta Q_T - \gamma Q_T^{2} - C_u(x_u) - P_s(x_{cg} - Q_d) - \lambda_1^{cg}(x_u \le \frac{1}{\gamma} (\beta - P_s - \gamma (x_{cg} - Q_d)))$$



To determine the optimum generation quantity of the host utility, we use the concept of active and inactive constraints (Luenberger 2003). We first determine the optimal x_u by considering the case where x_u is strictly less than the bound specified in equation (24). In this case, the host utility's profit maximization problem is a nonlinear unconstrained problem. The profit of the host utility π_{hu}^{cgp} , in problem P6 is nonlinear and concave w.r.t x_u . Therefore the optimal x_u can be determined from the first order necessary and sufficient conditions as follows

$$\frac{\partial \pi_{hu}^{cgp}}{\partial x_u} = \beta - 2\gamma \left(x_u + x_{cg} - Q_d \right) - m - 2nx_u = 0.$$
⁽²⁶⁾

$$x_{u}^{*_NB_nam} = \frac{\beta - 2\gamma(x_{cg} - Q_{d}) - m}{2(\gamma + n)}$$
(27)

where x_u^{*,NB_nam} is the non-binding optimal generation quantity of the electric utility if the reaction of the qualifying facility to the electricity price is to operate in the non-arbitrage mode (NAM).

To determine the condition under which the constraint will be binding we use the FONC for constrained optimization problem

$$\frac{\partial L_{hu}^{cgp}}{\partial x_u} = \beta - 2\gamma \left(x_u + x_{cg} - Q_d \right) - m - 2nx_u - \lambda_1^{cgp} = 0$$
⁽²⁸⁾

$$\frac{\partial L_{hu}^{ggp}}{\partial \lambda_1^{cgp}} = \frac{1}{\gamma} \left[\beta - P_s - \gamma \left(x_{cg} - Q_d \right) \right] - x_u = 0$$
⁽²⁹⁾

$$\lambda_1^{cgp}(x_u - \frac{1}{\gamma} \left[\beta - P_s - \gamma \left(x_{cg} - Q_d\right)\right]) = 0$$
(30)

$$\lambda_1^{cgp} \ge 0 \tag{31}$$



(25)

Equation (27) is the non-binding solution to problem P6. The optimal solution to problem P6 is binding with $x_u^{*,B_nam} = \frac{1}{\gamma} [\beta - P_s - \gamma (x_{cg} - Q_d)]$, when $\lambda_1^{cgp} \ge 0$. When the optimal solution is binding,

$$\lambda_1^{cgp} = \frac{2n\gamma x_{cg} + 2P_s(n+\gamma) - 2n\beta - \gamma(m+\beta) - \gamma Q_d 2n}{\gamma}$$
(32)

Equation (32) leads to a limit on the electricity demand Q_d of the thermal host based on which the optimal generation quantity x_u of the electric utility is such that $P_r \ge P_s$. Therefore the optimal generation of the electric utility in the Non-Arbitrage mode is given below,

$$x_{u}^{cgp_nam} = \begin{cases} \frac{\beta - m - 2\gamma(x_{cg} - Q_d)}{2(\gamma + n)}, \text{ when } Q_d > \frac{2n\gamma x_{cg} - 2n\beta - (m + \beta)\gamma + 2P_s(n + \gamma)}{2n\gamma} \\ \frac{1}{\gamma} \left[\beta - P_s - \gamma \left(x_{cg} - Q_d\right)\right], \text{ otherwise} \end{cases}$$
(33)

where $x_u^{cgp_nam}$ is the optimal generation quantity of the electric utility if the reaction of the qualifying facility to the electricity price is to operate in the non-arbitrage mode (NAM). *Arbitrage Mode* ($P_r < P_s$): In this case the profit maximization problem (P7) of the host utility is modified as follows,

$$\begin{aligned} \underset{x_u}{\overset{Max}{x_u}} \pi_{hu}^{cgp} &= \beta Q_T - \gamma Q_T^2 - C_u(x_u) - P_s x_{cg} + Q_d \left(\beta - \gamma Q_T\right) \\ \text{s.t.} \quad x_u &> \frac{1}{\gamma} [\beta - P_s - \gamma \left(x_{cg} - Q_d\right)] \end{aligned}$$
(34)

The problem P7 is a non-linear problem and the optimal x_u can be determined by using the

FONC to be

$$x_u^{*_NB_am} = \frac{\beta - 2\gamma x_{cg} + \gamma Q_d - m}{2(\gamma + n)}$$
(35)

where x_u^{*,NB_am} is the non-binding optimal generation quantity of the electric utility if the reaction of the qualifying facility to the electricity price is to operate in the arbitrage mode



(AM). Since the constraint in P7 is a strict inequality, the optimal non-binding solution in equation (35) should be strictly less than the upper bound specified in equation (34). The condition that ensures this is $\frac{1}{\gamma} (2n\beta + (m + \beta)\gamma - 2(n + \gamma)P_s + \gamma(2n + \gamma)Q_d - 2n\gamma x_{cg}) < 0$. Similar to the non-arbitrage case we can convert the condition to a limit on the electricity demand Q_d of the thermal host. Therefore the optimal generation optimal generation quantity x_u is such that $P_r < P_s$ is

$$x_u^{cgp_am} = \frac{\beta - m - 2\gamma x_{cg} + \gamma Q_d}{2(\gamma + n)}, \text{ when } Q_d < \frac{2n\gamma x_{cg} - 2n\beta - (m + \beta)\gamma + 2P_s(n + \gamma)}{\gamma(2n + \gamma)}$$
(36)

where $x_u^{cgp_am}$ is the optimal generation quantity of the electric utility if the reaction of the qualifying facility to the electricity price is to operate in the arbitrage mode (AM). Let

$$CN1 = \frac{2n\gamma x_{cg} - 2n\beta - (m+\beta)\gamma + 2P_s(n+\gamma)}{\gamma(\gamma+2n)} \text{ and } CN2 = \frac{2n\gamma x_{cg} - 2n\beta - (m+\beta)\gamma + 2P_s(n+\gamma)}{2n\gamma}.$$
 (37)

2.3.3 Equilibrium solution of CGP model

We next define the characteristics of the equilibrium solution to the Stackelberg game in the CGP configuration. An equilibrium solution of a game is the solution/strategy from which no player in the game has any incentive to deviate from. (Gibbons, 1992)

Definition of Stackelberg Equilibrium: The equilibrium solution to the Stackelberg game in the CGP configuration is defined as the set of decisions, $\{x_u^*, P_r^*, x_{cg}^*, y_{cg}^*, q_s^*, q_b^*\}$ that satisfy the following conditions

• The generation quantity x_u and the associated electricity price P_r maximizes the host utility's profit when she has a valid PURPA contract with a qualifying facility as part of which she trades electricity.



The cogeneration output of electricity x_{cg} and heat y_{cg}, electricity sold to the host utility q_s and electricity purchased from the host utility q_b for a given electricity price P_r maximizes the qualifying facility's profit and satisfies the electricity and heat demand of the thermal host

The CGP configurations Stackelberg game has six possible equilibrium solutions, only one of which will occur for a given set of parameters { α , β , γ , m, n, S_d , Q_d , c_1 , c_2 }. From equations (33) and (36) we get a mutually exclusive condition based on the parameter set that result in a unique equilibrium solution. Though there are six possible equilibrium solutions we focus on analyzing the four described below. This is because the behavior of the qualifying facility is the same as in the case of the non-arbitrage solutions described below. The four possible equilibrium solutions of interest are

Arbitrage/Binding solution: The qualifying facility cogenerates only the quantity of electricity that is required to satisfy the heat demand of the thermal host. (Binding: $x_{cg} = \alpha S_d$). The qualifying facility sells all the cogenerated electricity to the host utility and purchases electricity from the host utility to satisfy the electricity demand of the thermal host. This solution occur when the PURPA buyback price is lower than the marginal cost of generating at the level required to satisfy of the thermal host and the electricity demand of the thermal host is lower than CN1 a threshold value based on the remaining parameter values. *Arbitrage/Non-Binding solution:* The qualifying facility cogenerates more than the quantity of electricity that is required to satisfy the heat demand of the thermal host. (Non-Binding: $x_{cg} > \alpha S_d$). The qualifying facility sells all the cogenerated electricity to the host utility and purchases electricity from the host utility to satisfy the electricity demand of the thermal host.



This solution occurs when the PURPA buyback price is higher than the marginal cost of generating at the level required to satisfy the heat demand of the thermal host and the electricity demand of the thermal host is lower than CN1 a threshold value based on the remaining parameter values.

Non-Arbitrage/Binding solution: The qualifying facility cogenerates only the quantity of electricity that is required to satisfy the heat demand of the thermal host. (Binding: $x_{cg} =$ αS_d). The qualifying facility satisfies the electricity demand of the thermal host from the cogenerated electricity and sells only the surplus electricity to the host utility. This solution occurs when the PURPA buyback price is lower than the marginal cost of generating at the level required to satisfy the heat demand of the thermal host and the electricity demand of the thermal host is higher than CN2 a threshold value based on the remaining parameter values. *Non-Arbitrage/Non-Binding solution*: The qualifying facility cogenerates more than the quantity of electricity that is required to satisfy the heat demand of the thermal host. (Non-Binding: $x_{cg} > \alpha S_d$). The qualifying facility satisfies the electricity demand of the thermal host from the cogenerated electricity and sells only the surplus electricity to the host utility. This solution occurs when the PURPA buyback price is higher than the marginal cost of generating at the level required to satisfy the heat demand of the thermal host and the electricity demand of the thermal host is higher than CN2 a threshold value based on the remaining parameter values.

A summary of the equilibrium solution for the Stackelberg game between the electric utility and cogeneration facility is provided in Table 2.



Case	P_r^{cgp}	x_u^{cgp}	x ^{cgp}	
Non Binding Case				
$Q_d < CN1$	$P_r < P_s$	$\frac{\beta - m - 2\gamma x_{cg} + \gamma Q_d}{2(\gamma + n)}$	$\frac{P_s - c_1}{2c_2}$	
$CN1 \le Q_d < CN2$	$P_r = P_s$	$\frac{\left[\beta-P_s-\gamma(x_{cg}-Q_d)\right]}{\gamma}$	$\frac{P_s - c_1}{2c_2}$	
$Q_d > CN2$	$P_r > P_s$	$\frac{\beta - m - 2\gamma(x_{cg} - Q_d)}{2(\gamma + n)}$	$\frac{P_s - c_1}{2c_2}$	
Binding Case				
$Q_d < CN1$	$P_r < P_s$	$\frac{\beta - m - 2\gamma x_{cg} + \gamma Q_d}{2(\gamma + n)}$	αS_d	
$CN1 \le Q_d < CN2$	$P_r = P_s$	$\frac{\left[\beta-P_s-\gamma(x_{cg}-Q_d)\right]}{\gamma}$	αS_d	
$Q_d > CN2$	$P_r > P_s$	$\frac{\beta - m - 2\gamma(x_{cg} - Q_d)}{2(\gamma + n)}$	αS _d	
Constants	$CN1 = \frac{(2n\gamma x_{cg} - 2n\beta)}{2n\gamma x_{cg} - 2n\beta}$	$\frac{-(m+\beta)\gamma+2(n+\gamma)P_s}{\gamma(\gamma+2n)}; \text{CN2} = \frac{2n\gamma x_{cg}-2n\beta-1}{2n\gamma x_{cg}-2n\beta-1}$	$\frac{(m+\beta)\gamma+2P_{S}(n+\gamma)}{2n\gamma};$	
	$c_2 = \frac{c\alpha^2 + f\alpha + e}{\alpha^2}; \ c_1$	$=\frac{b\alpha+d}{\alpha};$		

 Table 2. Equilibrium solution(s) of the CGP configuration Stackelberg game

The consumer surplus corresponding to the equilibrium solution of the CGP model is given by

$$CS_{cgp} = v(S_d) + v(Q_d) + v(Q_T^{cgp}) - P_r(Q_T^{cgp})Q_T^{cgp}$$
(37)

 $v(S_d)$ and $v(Q_d)$ is the utility to the thermal host in consuming S_d MBtu of useful heat and Q_d MBtu of electricity. $v(S_d)$ and $v(S_d)$ are derived from the utility of the consumers who utilize the thermal host's products/process. Since $v(S_d)$ and $v(Q_d)$ will remain constant among all models their mathematical form is not explicitly specified in the paper. In all the four models, there are three types of participants – the retail electricity customers who are



consumers, the electric (host) utility who is a producer only and the qualifying facility unit which is both a producer (cogeneration facility) and a consumer (thermal host). In the consumer surplus expression we consider the consumer surplus of the retail electricity customers and the consumer surplus associated with the qualifying facility. In the CGP model the cost of generating S_d MBtu of useful heat and Q_d MBtu of electricity is accounted in the profit function of the qualifying facility. Hence the terms are not included in the consumer surplus expression in equation (37). $v(Q_T^{cgp})$ is the utility to the retail electricity customers in consuming Q_T^{cgp} MBtu of electricity. The utility to the retail electricity customers is the area under the inverse demand function between (0, Q_T^{cgp}).

$$v(Q_T^{cgp}) = \int_0^{Q_T^{cgp}} (\beta - \gamma x_u) \, dx_u = \beta x_u - \frac{\gamma x_u^2}{2} \Big]_0^{Q_T^{cgp}} = \beta Q_T - \frac{\gamma Q_T^{cgp^2}}{2}.$$

Therefore, $CS_{cgp} = v(S_d) + v(Q_d) + \frac{\gamma (Q_T^{cgp})^2}{2}.$ (38)

The total surplus associated with the CGP model is the sum of the producer surplus and the consumer surplus. In the CGP model the producer surplus is the profit to the host utility and the qualifying facility.

$$TS_{cgp} = CS_{cgp} + PS_{cgp} = CS_{cgp} + \pi_{hu}^{cgp} + \pi_{qf}^{cgp}$$
(39)

$$TS_{cgp} = v(S_d) + v(Q_d) + v(Q_T^{cgp}) - P_r^{cgp}(Q_T^{cgp})Q_T^{cgp} + P_r^{cgp}(Q_T^{cgp})Q_T^{cgp} - C(x_u^{cgp}) - P_sq_s + P_r^{cgp}(Q_T^{cgp})Q_d + P_sq_s - \bar{C}(x_{cg}^{cgp}) - P_r^{cgp}(Q_T^{cgp})Q_d$$
(40)

$$TS_{cgp} = v(S_d) + v(Q_d) + v(Q_T^{cgp}) - C_u(x_u^{cgp}) - \bar{C}_{cg}(x_{cg}^{cgp})$$
(40)



Illustrative Numerical Example:

An illustrative numerical example with hypothetical data is provided in this section. The parameter values used in the example are provided in Table 3.

Parameter	Numerical Value
P_s = PURPA buyback price at which the qualifying facility sells electricity to the	90
host utility (\$/MBtu)	
β = Vertical intercept of the inverse demand function faced by the utility	120
γ = Slope of the inverse demand function faced by the utility	0.02
α = Power to heat ratio of the cogeneration unit	0.68
a, b, c, d, e, $f = \text{cost coefficients of the cogeneration cost}$	250, 14.5, 0.03, 4.2,
	0.03, 0.03
l, m, n = cost coefficients of the host utility generation unit	1700, 7.8, 0.009
Q_d = Electricity demand of the thermal host (MBtu)	50
S_d = Heat demand of the thermal host (MBtu)	500

Table 3. Parameter Values used in the Numerical Example

Step 1: We first characterize the reaction function of the qualifying facility. To do this, we being by computing the value of unconstrained x_{cg} . We do this by inputting the values in the parameter table in $\frac{P_s-c_1}{2c_2}$ to obtain 249.09 MBtu. We next compute the value x_{cg} at its bound, αS_d which is 340 MBtu. From equation (15), we have that $x_{cg}^{cgp} = Max\{\frac{P_s-c_1}{2c_2}, \alpha S_d\}$ Therefore the optimal cogenerated electricity and heat are, $x_{cg}^{cgp} = 340$ MBtu and $y_{cg}^{cgp} =$



500 MBtu. From equation (18) we get the qualifying cogeneration facility's reaction function as follows,

$$q_s^{cg} = \begin{cases} 340 \text{ when } P_s > P_r \\ 240 \text{ when } P_s \le P_r \end{cases} \text{ and } q_b^{cg} = \begin{cases} 50 \text{ when } P_s > P_r \\ 0 \text{ when } P_s \le P_r \end{cases}$$

Since $x_{cg}^{cgp} = \frac{P_s - c_1}{2c_2} = 340$ the optimal generation plan of the qualifying facility is binding in the heat demand constraint, i.e. the cogeneration facility generates only the quantity of electricity whose associated heat will satisfy the heat demand of the thermal host. Step 2: We next compute the optimal generation plan for the host utility and the equilibrium solution to the Stackelberg game in the CGP model.

We calculate the value of CN1 and CN2 in equation (37) as follows,

$$CN1 = \frac{(2n\gamma x_{cg} - 2n\beta - (m+\beta)\gamma + 2(n+\gamma)P_s)}{\gamma(\gamma+2n)} = 824.2105 \text{ and}$$
$$CN2 = \frac{2n\gamma x_{cg} - 2n\beta - (m+\beta)\gamma + 2P_s(n+\gamma)}{2n\gamma} = 1740$$

From equations (33) and (36) we have that the unique equilibrium solution of the CGP model's Stackelberg game is based on the value of the electricity demand of the thermal host Q_d compared to CN1 and CN2. $Q_d = 50$ in the example, we have that $Q_d < \text{CN1}$, therefore the equilibrium solution of the Stackleberg game of the CGP model is the arbitrage-mode of operation for the qualifying facility and the generation quantity of the electric utility that will ensure that is, from Equation (36), $x_u^{cgp} = 1734.483$ (MBtu). The associated electricity price is $P_r^{cgp} = 79.510$ (\$/MBtu) and the profit to the host utility is $\pi_{hu}^{cgp} = 94262.48$ (\$). The qualifying facility's decisions at the equilibrium solution are given below $q_s^{cg} = 340$ MBtu,



 $q_b^{cg} = 50$ MBtu, $x_{cg}^{cgp} = 340$ MBtu and $y_{cg}^{cgp} = 500$ MBtu. The profit associated with the above decisions is $\pi_{qf}^{cgp} = 3002$ (\$).

2.4 Cogeneration facility as an Independent Power Producer (IPP) model

The most relevant benchmark of the cogeneration under PURPA (CGP) configuration is the cogeneration facility as an Independent Power Producer configuration (IPP). The IPP model is where the cogeneration facility sells electricity directly to the retail electricity customers. The IPP model is the most relevant benchmark due to the current partially de-regulated state of the electricity industry and the partial repeal of mandatory purchase requirement of PURPA. The number of NUG's that serve retail electricity customers has almost doubled in the last three years with 65 NUG's in 2004 to 116 NUG's in 2006 (EIA, 2006a). In addition, even though wholesale competition is considered a pre-requisite for retail competition in states such as Texas which have a combination of wholesale and retail competition with regulated monopolistic vertically integrated utilities, the options to the vertically integrated markets in terms of retail competition are being considered (AECT, 2007). Since many of the PURPA contracts that were signed in the early 80's are expiring and it is a critical time that the alternative options available to cogeneration facilities should be analyzed.

The electric utility is still the more dominant electricity supplier to the retail electricity customers. The IPP configuration is based on configurations that have come into existence after the de-regulation of the electricity industry and the opening of access to transmission and distribution services to NUG's. Examples of such an NUG is a 11.25 MW



cogeneration plant which supplies electricity and steam to 25 buildings in downtown Rochester MN including the Mayo Clinic, Rochester Methodist Hospital, Charter house and Sunstone Corporation Hotel. The plant is owned by Mayo Clinic and the Rochester Methodist Hospital. The majority of the city of Rochester, MN is serviced by the Rochester Public Utilities the local municipal utility. The Rochester Public Utilities is the largest municipal utility in Minnesota with over 45, 000 customers and a capacity of 193 MW (RPU, 2006). Another example is the Robert Muller Energy Center in Austin TX, a 4.5 MW gasfired cogeneration plant that serves the Dell Children's Hospital with electricity and steam. It exports its power to the grid. The owned and operated by the Austin Energy, the local electric utility and is paid the prevailing electricity rates.

In the IPP model the electric utility and the cogeneration facility are engaged in a Stackelberg game with the electric utility as the leader and the cogeneration facility as the follower. This is because even though the IPP's supply electricity to retail customers the number of customers they serve is much smaller compared to the local utility. Hence the price they charge and the quantity they supply will be heavily influenced by the local utility. Since we were unable to identify the exact mechanism by which the retail prices are set by these IPP's we assume that they take the price set by the local utility and determine the quantity they wish to supply. Hence the IPP model is similar to the CGP model with the critical difference that the price at which the cogeneration facility sells electricity is determined by the electric utility. The energy generation system configuration is shown in Figure 3.



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Figure 3. Cogeneration facility as an Independent Power Producer (IPP) configuration model

2.4.1 Cogeneration facility's generation planning problem

The cogeneration facility's generation planning problem in the IPP configuration is the profit maximization problem given below

$$\frac{Max}{x_{cg}}\pi_{cg}^{ipp} = P_r(x_{cg} - Q_d) - C_{cg}(x_{cg}, y_{cg})$$
(41)

s.t
$$y_{cg} \ge S_d$$
 (42)

$$x_{cg} = \alpha y_{cg} \tag{43}$$

Similar to the CGP model the decision variable y_{cg} can be eliminated using equation (43) to obtain the following profit maximization problem,

$$\frac{Max}{x_{cg}}\pi_{cg}^{ipp} = P_r(x_{cg} - Q_d) - \bar{C}_{cg}(x_{cg}) \qquad \text{s.t} \qquad x_{cg} \ge \alpha S_d$$
(44)

where $\overline{C}_{cg}(x_{cg})$ is the modified cost of cogeneration, the same as in the CGP model. The optimal solution to the cogeneration facility's profit maximization problem can be obtained



from the FONC using the concept of active and inactive constraints. The unconstrained solution to the cogeneration facility's generation planning problem is given below

$$x_{cg}^{*_NB} = \frac{P_r - c_1}{2c_2};$$
(45)

where $x_{cg}^{*,NB}$ is the unconstrained optimal cogenerated electricity in the IPP model. The optimal binding solution to the cogeneration facility's profit maximization problem is $x_{cg}^{*,B} = \alpha S_d$. Therefore the optimal cogenerated electricity in the IPP model is $x_{cg}^{ipp} = Max\{\frac{P_r - c_1}{2c_2}, \alpha S_d\}$ (46)

Reaction function of the cogeneration facility: In the IPP model the optimal cogenerated electricity is a function of the electricity price, P_r . Therefore the amount of electricity that the cogeneration facility will supply to the retail electricity customers for a given P_r is

$$S_{cg}(P_r) = x_{cg}^{ipp}(P_r) - Q_d.$$
 (47)

where $S_{cg}(P_r)$ is the reaction function of the cogeneration facility to the electricity price, P_r .

2.4.2 Electric utility's generation planning problem

The optimal generation quantity of the electric utility depends on the residual demand. From the inverse demand function, the total electricity demanded by the retail electricity customers as a function of electricity price is $Q_T^{ipp}(P_r) = \frac{1}{\gamma}(\beta - P_r)$. Of the total electricity demanded, $S_{cg}(P_r)$ is supplied by the cogeneration facility and the residual demand is satisfied by the electric utility. The residual demand for the electric utility as a function of the electricity price is given below

$$R(P_r) = Q_T^{ipp}(P_r) - S_{cg}(P_r)$$
(48)



The electric utility being the Stackelberg leader will determine the residual demand and associated electricity price that would maximize its profit. The reaction function of the cogeneration facility to an electricity market price determines the residual demand function of the electric utility. The reaction of the cogeneration facility is discontinuous with the discontinuity occurring when $P_r > c_1 + 2c_2\alpha S_d$. If $P_r > c_1 + 2c_2\alpha S_d$, the reaction of the cogeneration facility is a function of the electricity price. This is because $x_{cg}^{ipp}(P_r)$ is a function P_r and by equation (47) we know that the reaction of the cogeneration facility is its optimal generation plan for a given electricity price. If $P_r \leq c_1 + 2c_2\alpha S_d$ then the cogeneration facility's reaction is to generate a fixed quantity of electricity αS_d which leads to a fixed supply of $\alpha S_d - Q_d$ MBtu of electricity to the retail electricity customers. This leads to two cases – case (a) when $x_{cg}^{ipp}(P_r) = \frac{P_r - c_1}{2c_2}$ and case (b) when $x_{cg}^{ipp}(P_r) = \alpha S_d$

Case (a):
$$x_{cg}^{ipp}(P_r) = \frac{P_r - c_1}{2c_2}$$

When $x_{cg}^{ipp}(P_r) = \frac{P_r - c_1}{2c_2}$, the electricity supplied by the cogeneration facility to the retail electricity customers is $S_{cg}(P_r) = \frac{P_r - c_1}{2c_2} - Q_d$. Therefore the residual demand faced by the electric utility is

$$R_u(P_r) = \frac{1}{\gamma}(\beta - P_r) - \left(\frac{P_r - c_1}{2c_2} - Q_d\right)$$
(49)

From equation (49) we determine the inverse residual demand function faced by the electric utility as

$$P_r(x_u) = \frac{2c_2\beta - \gamma(2c_2(x_u - Q_d) - c_1)}{2c_2 + \gamma}$$
(50)



We know that the reaction of the cogeneration facility occurs when $P_r > c_1 + 2c_2\alpha S_d$. From equation (50) we have that the electricity price, P_r is a function of the generation quantity of the electric utility. Therefore the condition for the change in the reaction function of the cogeneration facility yields the following condition on the generation quantity x_u

$$\frac{1}{\gamma}(\beta - \gamma(\alpha S_d - Q_d) - c_1 - 2c_2\alpha S_d) > x_u$$
(51)

The generation planning problem of the electric utility when the reaction of the cogeneration facility is $x_{cg}^{ipp}(P_r) = \frac{P_r - c_1}{2c_2}$, in the IPP model is the profit maximization problem (P8) given below,

$$\frac{Max}{x_u} \pi_{eu}^{ipp} = \left(\frac{2c_2\beta - \gamma(2c_2(x_u - Q_d) - c_1)}{c_2 + \gamma}\right) x_u - l - mx_u - nx_u^2$$
(52)

s.t
$$x_u < \frac{1}{\gamma} (\beta - \gamma (\alpha S_d - Q_d) - c_1 - 2c_2 \alpha S_d)$$
 (53)

Problem P8 is a non-linear optimization problem. Solving for the optimal solution using the FONC since, π_{eu}^{ipp} is concave w.r.t x_u (Proof in Appendix A)

$$\frac{\partial \pi_{eu}^{ipp}}{\partial x_u} = \frac{2c_2\beta - \gamma(2c_2(x_u - Q_d) - c_1)}{2c_2 + \gamma} - m - 2nx_u - \frac{2\gamma c_2}{2c_2 + \gamma}x_u = 0$$
(54)

$$x_{u}^{ipp-NB} = \frac{2c_{2}(\beta + 2c_{2}Q_{d} - m) + \gamma(c_{1} - m)}{2(n\gamma + 2(n+\gamma)c_{2})}$$
(55)

where x_u^{ipp-NB} is the unconstrained optimal generation quantity of the electric utility in the IPP configuration if the reaction of the cogeneration facility is $x_{cg}^{ipp}(P_r) = \frac{P_r - c_1}{2c_2}$. Since the constraint specified by equation (53) is a strict inequality for equation (49) to be the

optimal solution it has to satisfy the following condition

$$\frac{2c_2(\beta + 2c_2Q_d - m) + \gamma(c_1 - m)}{2(n\gamma + 2(n+\gamma)c_2)} < \frac{1}{\gamma} (\beta - \gamma (\alpha S_d - Q_d) - c_1 - 2c_2\alpha S_d$$
(56)



$$2n\gamma c_{1} + \gamma^{2}c_{1} + 4c_{1}c_{2}(n+\gamma) - m(\gamma^{2} + 2\gamma c_{2}) - \beta(2n\gamma + 2\gamma c_{2} + 4nc_{2}) - Q_{d}(2n\gamma^{2} + 4n\gamma c_{2} + 2\gamma 2c_{2} + \alpha Sd2n\gamma 2 + 4\gamma 2c_{2}4n\gamma c_{2} + 8c_{2}2n+\gamma < 0$$

The condition on the electricity demand of the thermal host that ensures that the optimal generation quantity in equation (53) satisfies condition (56) is

$$\frac{2n\gamma c_{1}+\gamma^{2}c_{1}+4c_{1}c_{2}(n+\gamma)-m(\gamma^{2}+2\gamma c_{2})-\beta(2n\gamma+2\gamma c_{2}+4nc_{2})+\alpha S_{d}(2n\gamma^{2}+4\gamma^{2}c_{2}4n\gamma c_{2}+8c_{2}^{2}(n+\gamma))}{(2n\gamma^{2}+4n\gamma c_{2}+2\gamma^{2}c_{2})} < Q_{d}$$
(57)

Case (b):
$$x_{cg}^{ipp}(P_r) = \alpha S_d$$

When, $x_{cg}^{ipp}(P_r) = \alpha S_d$ the electricity supplied by the cogeneration facility to the retail electricity customers is $S_{cg}(P_r) = \alpha S_d - Q_d$. Therefore the residual demand faced by the electric utility is

$$R_u(P_r) = \frac{1}{\gamma}(\beta - P_r) - (\alpha S_d - Q_d)$$
(58)

From equation (58) we get the inverse residual demand function faced by the electric utility as

$$P_w(x_u) = \beta - \gamma(x_u + \alpha S_d - Q_d)$$
⁽⁵⁹⁾

The generation planning problem of the electric utility when the reaction of the cogeneration facility is $x_{cg}^{ipp}(P_r) = \alpha S_d$, in the IPP model is the profit maximization problem (P9) given below,

$$\begin{array}{l}
\underset{x_u}{\text{Max}} \pi_{eu}^{ipp} = (\beta - \gamma (x_u + \alpha S_d - Q_d)) x_u - C_u(x_u)
\end{array}$$
(60)

s.t
$$x_u \ge \frac{1}{\gamma} (\beta - \gamma (\alpha S_d - Q_d) - c_1 - 2c_2 \alpha S_d)$$
 (61)

The generation planning problem described above is a constrined non-linear optimization problem. The solution to the problem is obtained using the method of active and inactive



constraints. If we consider the constraint (61) inactive at optimality, problem P9 becomes a unconstrained non-linear problem. The optimal solution to the problem can be found from its FONC of the unconstrained non-linear problem as

$$\frac{\partial \pi_{eu}^{ipp}}{\partial x_u} = \beta - m - 2nx_u - \gamma x_u - \gamma (x_u + \alpha S_d - Q_d) = 0$$
(62)

$$x_u^{ipp-B-NB} = \frac{\beta - m - \gamma(\alpha S_d - Q_d)}{2(n+\gamma)}$$
(63)

where $x_u^{ipp-B-NB}$ is the unconstrained optimal generation quantity of the electric utility in the IPP configuration if the reaction of the cogeneration facility is $x_{cg}^{ipp}(P_r) = \alpha S_d$.

The constrained optimal generation quantity of the electric utility is in the IPP configuration if the reaction of the cogeneration facility is $x_{cg}^{ipp}(P_r) = \alpha S_d$ is

$$x_u^{ipp-B-NB} = \frac{1}{\gamma} (\beta - \gamma (\alpha S_d - Q_d) - c_1 - 2c_2 \alpha S_d)$$
(63)

The optimization problem P9 is a constrained non-linear problem can be obtained from the KKT conditions associated with the non-linear constrained problem. The Lagrangian function associated with the objective of problem P9 is

$$L_{eu}^{ipp} = (\beta - \gamma (x_u + \alpha S_d - Q_d))x_u - C_u(x_u) + \mu_1^{ipp}(x_u - \frac{1}{\gamma}(\beta - \gamma (\alpha S_d - Q_d) - c_1 - 2c_2\alpha S_d))$$
(64)

$$\frac{\partial L_{eu}^{ipp}}{\partial x_u} = \beta - m - 2nx_u - \gamma x_u - \gamma (x_u + \alpha S_d - Q_d) + \mu_1^{ipp} = 0$$
(65)

$$\frac{\partial L_{eu}^{ipp}}{\partial \mu_1^{ipp}} = x_u - \frac{1}{\gamma} \left(\beta - \gamma \left(\alpha S_d - Q_d \right) - c_1 - 2c_2 \alpha S_d \right) = 0$$
(66)

$$\mu_{1}^{ipp}\left(x_{u} - \frac{1}{\gamma}\left(\beta - \gamma\left(\alpha S_{d} - Q_{d}\right) - c_{1} - 2c_{2}\alpha S_{d}\right)\right) = 0 \; ; \; \mu_{1}^{ipp} \ge 0 \tag{67}$$

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If the optimal solution is constrained then $x_u^{ipp-B} = \frac{1}{\gamma} (\beta - \gamma (\alpha S_d - Q_d) - c_1 - 2c_2 \alpha S_d)$ and

$$\begin{split} \mu_1^{ipp} &\geq 0. \text{ From equation (65) we get} \\ \mu_1^{ipp} &= \frac{m\gamma + \beta(2n+\gamma) - 2nc_1 - 2\gamma c_1 + (2n\gamma + \gamma^2)Q_d + \alpha(-2n\gamma - \gamma^2 - 4nc_2 - 4\gamma c_2)S_d}{\gamma} \geq 0 \text{ . Therefore the} \\ \text{condition when the optimal solution is } x_u^{ipp-B-B} \text{ is} \\ m\gamma + \beta(2n+\gamma) - 2nc_1 - 2\gamma c_1 + (2n\gamma + \gamma^2)Q_d + \alpha(-2n\gamma - \gamma^2 - 4nc_2 - 4\gamma c_2)S_d \geq 0 \quad (68) \\ \text{Since the optimal solution is binding when } \mu_1^{ipp} \geq 0, \text{ the optimal solution will be non-binding when } \mu_1^{ipp} < 0. \text{ Therefore the condition under which the optimal solution of problem} \end{split}$$

P9 will be $x_u^{ipp-B-NB}$ is

$$m\gamma + \beta(2n+\gamma) - 2nc_1 - 2\gamma c_1 + (2n\gamma + \gamma^2)Q_d + \alpha(-2n\gamma - \gamma^2 - 4nc_2 - 4\gamma c_2)S_d < 0$$
(69)

2.4.3 Equilibrium solution of IPP model

We next define the characteristics of the equilibrium solution to the Stackelberg game in the IPP configuration. An equilibrium solution of a game is the solution/strategy from which no player in the game has any incentive to deviate from. (Gibbons, 1992)

Definition of Stackelberg Equilibrium: The equilibrium solution to the Stackelberg game in the IPP configuration is defined as the set of decisions, $\{x_u^*, P_r^*, x_{cg}^*, y_{cg}^*\}$ that satisfy the following conditions

• The generation quantity x_u and the associated electricity price P_r maximize the host utility's profit given the supply of electricity to the retail electricity customers by the cogeneration facility for the electricity.



• The cogeneration output of electricity x_{cg} and heat y_{cg} , maximizes the qualifying facility's profit for a given electricity price P_r and satisfies the electricity and heat demand of the thermal host.

Similar to the CGPE model the IPPE model has three possible solutions. Based on the set of given parameters { α , β , γ , m, n, S_d , Q_d , c_1 , c_2 }, we get conditions which determine which of the solutions is the Stackelberg equilibrium. The conditions are derived based on the Lagrangian multiplier associated with the constraints to the generation planning problem of the electric utility. The IPP configuration has three possible equilibrium solutions. We mainly focus on two solutions, non-binding solution and binding solution with $P_r < c_1 + 2c_2\alpha S_d$. This is because the behavior and the quantity of electricity supplied by the cogeneration facility is the same in the two possible binding solutions. A summary of the possible equilibrium solutions to the IPP model is provided below in Table 4.

Condition	x_u^{ipp}	x_{cg}^{ipp}	
Non-Binding;	$\frac{2c_2(\beta+2c_2Q_d-m)+\gamma(c_1-m)}{2c_2(\beta+2c_2Q_d-m)+\gamma(c_1-m)}$	$P_r - c_1$	
CN3 < 0	$2(n\gamma+2(n+\gamma)c_2)$	2 <i>c</i> ₂	
Binding with	$\beta - \gamma(\alpha S_d - Q_d) - c_1 - 2c_2\alpha S_d$	αS_d	
$P_r = c_1 + 2c_2\alpha S_d ;$	γ		
$CN4 \ge 0$			
Binding with	$\beta - m + \gamma (Q_d - \alpha S_d)$	αS_d	
$P_r < c_1 + 2c_2 \alpha S_d;$	$2(n+\gamma)$		
CN4 < 0			
Constants: $c_2 = \frac{c\alpha^2 + f\alpha + e}{\alpha^2}; \ c_1 = \frac{b\alpha + d}{\alpha};$			
$CN3 = 2(n\gamma^2 + 4n\gamma c_2 + 2\gamma^2 c_2 + 4nc_2^2 + 4\gamma c_2^2)\alpha S_d - m(\gamma^2 + 2\gamma c_2) - 2\beta(n\gamma + 2nc_2 + 2\beta c_2) - 2\beta(n\gamma + 2nc_2) - 2\beta(n\gamma + 2nc_2) - 2\beta(n\gamma + 2nc_2) - 2\beta(n\gamma + 2nc_2)$			
<i>yc2–y2+2yc2euPn+c1</i> 2 <i>ny+y2</i> +4 <i>nc2</i> +4 <i>yc2–2ny2</i> +2 <i>nyc2</i> +2 <i>y2c2Qd</i> ;			
$CN4 = m\gamma + \beta(2n + \gamma) - 2nc_1 - 2\gamma c_1 + (2n\gamma + \gamma^2)Q_d + \alpha(-2n\gamma - \gamma^2 - 4nc_2 - 4\gamma c_2)S_d;$			

Table 4. Equilibrium solution(s) of the IPP configuration Stackelberg game



The consumer surplus associated with the equilibirum solution of the Stackelberg game in the IPP configuration model is given below

$$CS_{ipp} = \nu(S_d) + \nu(Q_d) + \nu(Q_T^{ipp}) - P_r^{ipp}Q_T^{ipp}$$
(70)

Similar to the CGP model, $v(S_d)$ and $v(Q_d)$ are considered to be constant and the utility to the retail electricity consumers is obtained from the inverse demand function. The consumer surplus expression is similar to the CGP model since the cost to the thermal host of satifying the heat and electricity demand consumed is accounted for in the profit function of the cogeneration facility rather than in the consumer surplus.Therefore the consumer surplus of the IPP model is given below

$$CS_{ipp} = v(S_d) + v(Q_d) + \frac{\gamma(Q_T^{ipp})^2}{2} \quad \text{with } Q_T^{ipp} = x_u^{ipp} + x_{cg}^{ipp} - Q_d$$
(71)

The total surplus associated with the equilibirum solution of the IPP model is

$$TS_{ipp} = CS_{ipp} + PS_{ipp} = CS_{ipp} + \pi_{eu}^{ipp} + \pi_{cg}^{ipp} = v(S_d) + v(Q_d) + v(Q_T^{ipp}) - P_w^{ipp}(x_u^{ipp})Q_T^{ipp} + P_w^{ipp}(x_u^{ipp})(x_{cg}^{ipp} - Q_d) - \bar{C}_{cg}(x_{cg}^{ipp}) + P_w^{ipp}(x_u^{ipp})x_u^{ipp} - C_u(x_u^{cgs})$$

Therefore the total surplus in the IPP model is

$$TS_{ipp} = v(S_d) + v(Q_d) + v(Q_T^{ipp}) - \bar{C}_{cg}(x_{cg}^{ipp}) - C_u(x_u^{ipp})$$
(72)

Illustrative Numerical Example: Based on the numerical values provided in Table 3 a illustrative numerical example is shown below.

Step1 : Calculate the value of the constants CN3 and CN4 to be 0.64928 qnd -2.18148 respectively. Since CN3 > 0 and CN4 < 0 the equilibirum solution is the binding solution with $P_r < c_1 + 2c_2\alpha S_d$. The generation quantity of the electric utility that is associated with



this equilibirum solution is $x_u^{ipp} = \frac{\beta - m + \gamma(Q_d - \alpha S_d)}{2(n+\gamma)} = 1843.483$. The associated electricity price $P_r^{ipp} = 77.51$ (\$/MBtu). The profit to the electric utility is \$9754.483. Step 2: Based on the electricity price, the cogeneration facility estimates, $\frac{P_r^{ipp} - c_1}{2c_2} =$ 204.4434. The generation level required to satisfy the heat demand of the thermal host is $\alpha S_d = 340$. Since the optimal generation plan for the cogeneration facility is $x_{cg}^{ipp} =$ $Max\{\frac{P_r^{ipp} - c_1}{2c_2}, \alpha S_d\}$, the $x_{cg}^{ipp} = 340$. The cogeneration facility satisfies the electricity demand of the thermal host and sells the remaining , 340 -50 MBtu = 290 MBtu to retail electricity customers at the prevailing electricity price. The associated profit to the cogeneration facility is \$3255.517.

2.5 Self-Generation with cogeneration (SCG) model

In the SCG model, the thermal host's heat demand and electricity demand is satisfied by the cogeneration facility. The cogeneration facility and the thermal host have no interaction with the electric utility or the retail electricity customers. The energy generation system configuration is shown in Figure 4. The total cost, C_T^{scg} , of satisfying the heat and electricity demand of the thermal host is $\overline{C}_{cg}(\alpha S_d) = a + c_1 \alpha S_d + c_2 \alpha^2 S_d^2$ which is the cost of cogenerating exactly the quantity of heat and electricity required to satisfy the just the heat demand of the thermal host.





Figure 4. Self generation with cogeneration (SCG) configuration model

The electric utility's generation planning problem in the SCG is given below

$$Max \pi_{eu}^{scg} = P_r(x_u) x_u - C_u(x_u)$$
(73)

In the SCG model the inverse demand function of the retail electricity customers and the generation cost function of the electric utility retain the same functional form as in the CGP and IPP models. The optimal generation quantity of the electric utility in the SCG model is obtained from its FONC as

$$x_u^{scg} = \frac{\beta - m}{2(n + \gamma)} \tag{74}$$

The consumer surplus in the SCG model is given by the following expression

$$CS_{scg} = v(S_d) + v(Q_d) + v(x_u^{scg}) - P_r(x_u^{scg}) x_u^{scg} - \overline{C}_{cg}(\alpha S_d)$$
(75)

Similar to the CGP and IPP models we estimate $v(x_u^{cg})$ from the inverse demand function of the retail electricity customers and obtain the consumer surplus to be

$$CS_{cg} = v(S_d) + v(Q_d) + \frac{\gamma}{2} x_u^{cg^2} - \overline{C}_{cg} (\alpha S_d)$$
(76)



The total surplus corresponding to the SCG model is the sum of the consumer surplus and the profit of the electric utility

$$TS_{cg} = CS_{cg} + \pi_{eu}^{cg} = v(S_d) + v(Q_d) + v(Q_T^{cg}) - C_u(x_u^{cg}) - \overline{C}_{cg}(x_{cg}^{cg})$$
(77)

2.6 Heat Production without cogeneration (HP) model

In the HP model, the thermal host utilizes a conventional heat/steam generator to satisfy its heat demand and purchases electricity from the electric utility to satisfy its electricity demand. The energy generation system configuration is shown in Figure 5.





The thermal host does not gain revenue due to electricity sales but incurs the cost of satisfying its energy needs. The total cost, C_T^{hp} of satisfying the heat and electricity demand of the thermal host is given by $C_T^{hp} = C_h(S_d) + P_r^{hp}Q_d = i + jS_d + kS_d^2$. $C_h(S_d) = i + jS_d + kS_d^2$ is the quadratic cost function of a conventional boiler.



The electric utility's generation planning model is the profit maximization problem P10 given below

$$\max_{x_u} \pi_{eu}^{hp} = P_r(Q_T)Q_T - C_u(x_u) + P_r(Q_T)Q_d \quad \text{with } Q_T = x_u - Q_d$$
(78)

 Q_T is the total electricity supply by the host utility to the retail electricity customers. The inverse demand function and the generation cost function for the electric utility remains the same as in the CGP and IPP models. Since problem P10 is a non-linear unconstrained problem the optimal generation quantity for the electric utility in the HP model can be obtained from its FONC as

$$x_u^{hp} = \frac{\beta - m + \gamma Q_d}{2(n + \gamma)} \tag{79}$$

The consumer surplus in the HP model is

$$CS_{hp} = v(S_d) + v(Q_d) + v(Q_T^{hp}) - P_r(Q_T^{hp})Q_T^{hp} - C_T^{hp}$$
(80)

Similar to the CGP model the utility in consuming Q_T^{hp} MBtu of electricity to the retail electricity customers is the area under the inverse demand function. Unlike the CGP model, the cost of satisfying S_d MBtu of useful heat and Q_d MBtu of electricity for the service utility is included in the consumer surplus expression of the HP model. This is because in the HP model, the thermal host is only a consumer of electricity and heat and does not supply any energy products as in the case of the CGP model. Therefore,

$$CS_{hp} = v(S_d) + v(Q_d) + \frac{\gamma}{2}Q_T^{hp2} - C_T^{hp}$$
(81)



The total surplus is the sum of the consumer surplus and the producer surplus. In the HP model the producer surplus includes the profit to the electric utility and the consumer surplus corresponding to the consumption of the thermal host and the retail electricity customers.

$$TS_{hp} = v(S_d) + v(Q_d) + v(Q_T^{hp}) - C_u(x_u^{hp}) - C_h(S_d)$$
(82)

2.7 Economic performance of Cogeneration under PURPA w.r.t. the benchmarks

In this section of the paper, we evaluate the economic performance of PURPA by comparing the total surplus in the CGP model with the three benchmarks – HP model, SCG model and IPP model. We use total surplus as the measure of economic welfare performance since it accounts for the economic benefits to all the participants in the system including the retail electricity customers. If we only consider the profits to the electric utility and the cogeneration facility, the economic performance evaluation will not give a completer picture. Also since PURPA is a result of government intervention and the government will be more interested in maximizing total surplus rather than individual participant profits, the total surplus as a performance criteria for economic welfare is appropriate.

2.7.1 Comparison of total surplus in CGP and IPP configurations

To explicitly understand the benefits of PURPA we need to compare the cogeneration facility as an Independent Power Producer (IPP) model and the Cogeneration under PURPA (CGP) model. Since one of the intents of PURPA is to encourage generation and sale of electricity from non-utility electricity producers that are more fuel efficient, PURPA stipulates that



electric utility's have to purchase power at a pre-determined, fixed rate, P_s . This fixed rate is called the PURPA buyback rate (Joskow and Jones, 1983; Rose and NcDonald, 1991). P_s is set by the regulating authority of the state where the host utility and qualifying facility are located, on a case by case basis and is equivalent to the avoided cost of the host utility. The avoided cost is defined as the cost the electric utility avoids by not purchasing or generating the quantity of cogenerated electricity sold to it by the cogeneration facility. The intent of PURPA was to provide the cogeneration facilities and other qualifying facilities a captive customer in the form of the electric utility. The reason the buyback rate is the electric utility's avoided cost was to make the transaction cost neutral to the electric utility and the wholesale/retail electricity markets (Hirsh, 1991).

In the IPP configuration model, the generation and sale of electricity from cogeneration facilities and other non-utilities is possible due to the de-regulation of the electricity industry. In the IPP configuration, by ensuring that local utilities provide fair access to transmission and distribution services to cogeneration facilities, the participation of non-utility generators in ensured in retail and wholesale markets (where they exist). The main difference between the IPP and CGP models is the price at which the cogeneration facility sells electricity. In the CGP model the price is set by a regulatory mechanism while in the IPP model the price is set by the game between the cogeneration facility and the electric utility. By comparing the total surplus in these two models we will be able to evaluate the economic performance of cogeneration under PURPA



The IPP model has three possible equilibrium solutions –binding solution with $P_r^{ipp} = c_1 + 2c_2\alpha S_d$, binding solution with $P_r^{ipp} < c_1 + 2c_2\alpha S_d$ and non-binding solution with $P_r^{ipp} > c_1 + 2c_2\alpha S_d$.

Case (a): Binding solution with $P_r^{ipp} < c_1 + 2c_2 \alpha S_d$;

If the equilibrium solution of the IPP model is the binding solution with $P_r^{ipp} < c_1 + 2c_2\alpha S_d$ then $x_{cg}^{ipp} = \alpha S_d$ and $x_u^{ipp} = \frac{\beta - m - \gamma(\alpha S_d - Q_d)}{2(n+\gamma)}$. From equations (40) and (77) we see that the total surplus expressions are very similar and they differ only the utility to the retail electricity customers, the cost of generation to the electric utility and the cost of cogeneration to the cogeneration facility.

The utility to the retail electricity customers is a function of the total electricity supplied to them. In the CGP model the total electricity supplied to the retail customers is as follows,

$$Q_T^{cgp} = \frac{\beta - m}{2(n+\gamma)} + \frac{2n(x_{cg}^{cgp} - Q_d)}{2(n+\gamma)}$$
(Non-arbitrage mode) or
$$Q_T^{cgp} = \frac{\beta - m}{2(n+\gamma)} - \frac{\gamma Q_d}{(n+\gamma)} + \frac{2n(x_{cg}^{cgp} - Q_d)}{2(n+\gamma)}$$
(Arbitrage mode) (83)

The total supply of electricity to the retail customers in the IPP model, when $x_{cg}^{ipp} = \alpha S_d$ and

when
$$x_u^{ipp} = \frac{\beta - m - \gamma(\alpha S_d - Q_d)}{2(n+\gamma)}$$
, is

$$Q_T^{ipp} = \frac{\beta - m}{2(n+\gamma)} + \frac{(2n+\gamma)(\alpha S_d - Q_d)}{2(n+\gamma)}$$
(84)

If the equilibrium solution of the CGP configuration is arbitrage mode binding solution then, $x_{cg}^{cgp} = \alpha S_d$. From equations (83), and (84) we have that, $Q_T^{ipp} > Q_T^{cgp}$ and the



utility of the retail electricity customers is greater in the IPP model than the CGP model i.e. $v(Q_T^{ipp}) > v(Q_T^{cgp}).$

The cost of generation for the electric utility is less in the CGP than in the IPP model since from equations (28) and (44), $x_u^{ipp} > x_u^{cgp}$ thereby reducing the cost of generation in the CGP model. The cost of cogeneration is the same in both the CGP and IPP models. Therefore the condition under which the CGP model results in greater total surplus than in the IPP model is

$$\nu(Q_T^{ipp}) - \nu(Q_T^{cgp}) < C_u(x_u^{ipp}) - C_u(x_u^{cgp})$$
(85)

The condition (85) can be written mathematically as

$$\beta (Q_T^{ipp} - Q_T^{cgp}) - \frac{\gamma}{2} (Q_T^{ipp^2} - Q_T^{cgp^2}) < m(x_u^{ipp} - x_u^{cgp}) + n(x_u^{ipp^2} - x_u^{cgp^2})$$

Simplifying the above expression (details in Appendix B) we get

$$\beta + \gamma Q_d - m < (\frac{\gamma}{2} - n)\alpha S_d \tag{86}$$

Since $x_u^{ipp} > 0$, we have $\beta + \gamma Q_d - m > \gamma \alpha S_d$. Therefore the condition (86) will not occur. Hence if the equilibrium solution of the CGP model is the arbitrage-binding solution and the equilibrium solution of the IPP model is such that the cogeneration facility only generates electricity to satisfy the thermal host's heat demand, the CGP configuration does worse in terms of total surplus than IPP configuration. The results of the comparison are summarized in Table 4. (Details in the Appendix B)



Criteria	CGP	IPP	Condition
Total Surplus		Higher	
Consumer Surplus		Higher	
-		-	
Profit of cogeneration facility	Higher		$P_{\rm s} > P_{\rm r}^{ipp}$
	-		
Profit of electric utility		Higher	

Table 5: Comparison of economic performance between the CGP and IPPconfiguration when equilibrium solution in CGP is arbitrage/binding and equilibriumsolution in IPP is IPP binding with $P_r^{ipp} < c_1 + 2c_2\alpha S_d$.

If the equilibrium solution of the CGP configuration is arbitrage mode non-binding solution then, $x_{cg}^{cgp} = \frac{P_s - c_1}{2c_2} > \alpha S_d$. If $\alpha S_d + Q_d < \frac{2n\Delta}{\gamma}$ with $\Delta = \frac{P_s - c_1}{2c_2} - \alpha S_d$ then $Q_T^{cgp} > Q_T^{ipp}$ and the utility to the retail electricity customers will be greater in the CGP configuration than in the IPP configuration. The cost of generation for the electric utility is less in the CGP than in the IPP model since from equations (28) and (44), $x_u^{ipp} > x_u^{cgp}$ thereby reducing the cost of generation in the CGP model. Since in case of the non-binding equilibrium solution to the CGP model, $x_{cg}^{cgp} = \frac{P_s - c_1}{2c_2} > \alpha S_d$, the cost of cogeneration will increase. This leads to the following condition for the total surplus in the CGP model is to be greater than the total surplus in the IPP model.

$$\nu(Q_T^{cgp}) - \nu(Q_T^{ipp}) + C_u(x_u^{ipp}) - C_u(x_u^{cgp}) < \bar{C}_{cg}(x_{cg}^{cgp}) - \bar{C}_{cg}(x_{cg}^{ipp})$$
(87)

The condition (87) states that if sum of the gain in utility to the retail electricity market customers in the CGP model is less and the gain to the electric utility due to reduced generation cost is less than the additional cost to the electric utility in generating that



additional electricity, then the total surplus in the CGP configuration will be greater than the total surplus in the IPP configuration.

If the equilibrium solution of the CGP configuration is non-arbitrage mode binding solution then, x_{cg}^{cgp} and $x_u^{cgp} = \frac{\beta - m - 2\gamma(\alpha S_d - Q_d)}{2(n+\gamma)}$. Since Q_T^{ipp} and Q_T^{cgp} differ only in the quantity of electricity that the electric utility supplies and $x_u^{ipp} > x_u^{cgp}$ and $Q_T^{ipp} > Q_T^{cgp}$. Hence the condition under which the total surplus of the CGP configuration will exceed the total surplus of the IPP configuration is the same as equation (82). In case of the nonarbitrage solution the condition in equation (82) simplifies to

$$\left(\frac{\gamma}{2} - n\right)\left(\alpha S_d - Q_d\right) > \beta - m \tag{88}$$

Since $x_u^{ipp} > 0$, we have $\beta - m > \gamma(\alpha S_d - Q_d)$. Therefore the condition in equation (88) will never be satisfied. Hence if the equilibrium solution in the CGP model is the non-arbitrage binding solution, the total surplus in the CGP model will be lower than the total surplus in the IPP configuration. The results of the comparison are summarized in Table 5.

Criteria	CGP	IPP
Total Surplus		Higher
Consumer Surplus		Higher
Profit of cogeneration facility		Higher
Profit Surplus of electric utility	Higher	

Table 6: Comparison of economic performance between the CGP and IPP configuration when equilibrium solution in CGP is non-arbitrage/binding solution and the equilibrium solution in the IPP configuration is IPP binding with $P_r^{ipp} < c_1 + 2c_2\alpha S_d$.



If the equilibrium solution of the CGP configuration is non-arbitrage mode nonbinding solution then, $x_{cg}^{cgp} = \frac{P_s - c_1}{2c_2} > \alpha S_d$. If $\alpha S_d < \frac{n\Delta}{n+\gamma}$ then with $\Delta = \frac{P_s - c_1}{2c_2} - \alpha S_d$, $Q_T^{cgp} > Q_T^{ipp}$ and the utility to the retail electricity customers will be greater in the CGP configuration than in the IPP configuration. The cost of generation for the electric utility is less in the CGP than in the IPP model since from equations (28) and (44), $x_u^{ipp} > x_u^{cgp}$ thereby reducing the cost of generation in the CGP model. Since in case of the non-binding equilibrium solution to the CGP model, $x_{cg}^{cgp} = \frac{P_s - c_1}{2c_2} > \alpha S_d$, the cost of cogeneration will increase. This leads to the following condition for the total surplus in the CGP model is to be greater than the total surplus in the IPP model.

$$\nu(Q_T^{cgp}) - \nu(Q_T^{ipp}) + C_u(x_u^{ipp}) - C_u(x_u^{cgp}) < \bar{C}_{cg}(x_{cg}^{cgp}) - \bar{C}_{cg}(x_{cg}^{ipp})$$
(89)

It should be noted that the above condition is the same as in the case of the arbitrage mode non-binding solution.

2.7.2 Comparison of total surplus in CGP and HP configurations

From equations (40) and (82) we that

$$TS_{cgp} = v(S_d) + v(Q_d) + v(Q_T^{cgp}) - C_u(x_u^{cgp}) - \bar{C}_{cg}(x_{cg}^{cgp})$$
$$TS_{hp} = v(S_d) + v(Q_d) + v(Q_T^{hp}) - C_u(x_u^{hp}) - C_h(S_d)$$

 TS_{cgp} and TS_{hp} differ in the third, fourth and fifth terms only. The third and fouth terms of the total surplus expression are the economic utility to the retail electricity customers and the cost of generation to the electric utility. The fifth term in TS_{hp} is the cost of generating S_d (MBtu) using a conventional boiler while in TS_{cgp} it is the cost of cogenerating x_{cg}^{cgp} MBtu



of of electricity and αx_{cg}^{cgp} (MBtu) of heat. In comparing the total surplus for the HP and CGP models we have to compare the four possible equilibrium solutions of the CGP model – arbitrage/binding solution, arbitrage/non-binding solution, non-arbitrage/binding solution and non-arbitrage/non-binding solution with the optimal solution of the HP model. The occurrence of one of the equilibrium is dependent on the set of parameters,

$$\{\alpha, \beta, \gamma, m, n, c_1, c_2, S_d, Q_d\}$$

The total electricity supply Q_T , to the retail electricity customers is greater in the CGP model than in the HP model. Since the supply is greater in the CGP model the utility to the retail electricity customers is also greater in the CGP model. This is mathematically shown as follows. The total electricity supply to the retail electricity customers in the HP configuration is given by

$$Q_T^{hp} = x_{cg}^{hp} - Q_d$$
 with $x_{cg}^{hp} = \frac{\beta - m + \gamma Q_d}{2(n+\gamma)}$.

From equation (83) we can see that

$$Q_T^{cgp} = \frac{\beta - m}{2(n+\gamma)} + \frac{n(x_{cg}^{cgp} - Q_d)}{(n+\gamma)} = Q_T^{hp} + \frac{nx_{cg}^{cgp}}{(n+\gamma)} + \frac{\gamma Q_d}{2(n+\gamma)}$$
(Non-arbitrage mode) or
$$Q_T^{cgp} = \frac{\beta - m}{2(n+\gamma)} - \frac{\gamma Q_d}{(n+\gamma)} + \frac{n(x_{cg}^{cgp} - Q_d)}{(n+\gamma)} = Q_T^{hp} + \frac{nx_{cg}^{cgp}}{(n+\gamma)}$$
(Arbitrage Mode)

Since, $Q_T^{cgp} > Q_T^{hp}$, $v(Q_T^{cgp}) > (Q_T^{hp})$. Therefore the utility to the retail electricity customers is increased in the CGP model when compared to the utility to the retail electricity customers in the HP model. Also, from equation (38) and (80) we have that the consumer surplus of the CGP and HP configurations are $CS_{cgp} = v(S_d) + v(Q_d) + \frac{\gamma(Q_T^{cgp})^2}{2}$ and $CS_{hp} = v(S_d) + v(S_d$

 $v(Q_d) + \frac{\gamma(Q_T^{hp})^2}{2}$ respectively. The expressions differ only in the value of the total electricity



supply to the retail electricity customers. Since $Q_T^{cgp} > Q_T^{hp}$, $CS_{cgp} > CS_{hp}$ and the consumer surplus is greater in the CGP configuration (for any of the four possible equilibrium solutions) than in the HP configuration.

From equations (28) and (35) we have that

$$x_u^{cgp} = \frac{\beta - m + \gamma Q_d - 2\gamma x_{cg}^{cgp}}{2(n+\gamma)} = x_u^{hp} - \frac{\gamma x_{cg}^{cgp}}{(n+\gamma)}$$
(Non-Arbitrage Mode) or
$$x_u^{cgp} = \frac{\beta - m + 2\gamma Q_d - 2\gamma x_{cg}^{cgp}}{2(n+\gamma)} = x_u^{hp} - \frac{\gamma}{(n+\gamma)} (x_{cg}^{cgp} - \frac{Q_d}{2})$$
(Arbitrage Mode)

Therefore, the generation quantity of the electric utility is less the CGP model than in the HP model. This reduces the cost of generation to the electric utility i.e. $C_u(x_u^{cgp}) < C_u(x_u^{hp})$. The cost of cogeneration is generally higher than cost of generating process heat using a conventional boiler (Joskow and Jones, 1983). Hence the thermal host incurs additional cost in energy production in the CGP model than in the HP model. Therefore the condition under which the total surplus in CGP model will be greater than the total surplus in the HP model is given below

$$v(Q_T^{cgp}) - v(Q_T^{hp}) + C_u(x_u^{hp}) - C_u(x_u^{cgp}) < \bar{C}_{cg}(x_{cg}^{cgp}) - C_h(S_d)$$

$$= \frac{x_{cg}^{cgp}}{(n+\gamma)} \left[n\beta + \gamma m + \frac{n\gamma}{2} (Q_d + 3x_u^{hp}) - \frac{n\gamma(n+2\gamma)x_{cg}^{cgp}}{2(n+\gamma)} \right] > \bar{C}_{cg}(x_{cg}^{cgp}) - C_h(S_d)$$
(90)

Therefore cogeneration under PUPRA (CGP) will result in higher total surplus than heat production (HP) without cogeneration by the thermal host, if the additional cost of cogeneration incurred by the qualifying facility in the CGP model is less than the gains in total surplus due to the increased utility to retail electricity customers and the cost reduction to the electric utility.



2.7.3 Comparison of total surplus in CGP and SCG configurations

In the HP model there was no cogeneration and hence the comparison between the HP and CGP models helped identify the conditions under which cogeneration is advantageous to the society. But the previous comparison does not explicitly identify if cogeneration under PURPA is economically beneficial. One of the intents of PURPA is to promote non-utility electricity producers such as cogeneration facilities and small power producers (Hirsh, 1999). Especially those industrial and commercial cogeneration facilities that due to their very high heat demand and low electricity demand as modeled in this dissertation which will have excess electricity generated at minimal extra costs. In the SCG model the thermal host though attached to a cogeneration facility is not an independent power producer or qualifying facility. The cogeneration facility does not supply electricity to the retail electricity customers or to the electric utility. The SCG model helps identify the conditions under which cogeneration facilities being electricity suppliers is advantageous from a total surplus perspective.

From equations (40) and (77) we that

$$TS_{cgp} = v(S_d) + v(Q_d) + v(Q_T^{cgp}) - C_u(x_u^{cgp}) - \bar{C}_{cg}(x_{cg}^{cgp})$$
$$TS_{scg} = v(S_d) + v(Q_d) + v(x_u^{scg}) - C_u(x_u^{scg}) - \bar{C}_{cg}(\alpha S_d)$$

Since $v(S_d)$ and $v(Q_d)$ will remain constant among all models we focus on economic utility to the retail electricity customers, the cost of generation to the electric utility and the cost of cogeneration to the thermal host to compare the total surplus in the CG and SCG models. We once again compare the four possible equilibrium solutions of the CGP model – Arbitrage/binding solution, Arbitrage/non-Binding solution, Non-Arbitrage/binding solution


and Non-Arbitrage/non-Binding solution with the optimal solution of the SCG model. The occurrence of one of the equilibrium is dependent on the set of parameters,

$$\{\alpha,\beta,\gamma,m,n,c_1,c_2,S_d,Q_d\}.$$

The total electricity supply Q_T , to the retail electricity customers in the SCG model is x_{cg}^{scg} since the electric utility is their sole supplier $Q_T^{scg} = x_{cg}^{scg} = \frac{\beta - m}{2(n+\gamma)}$.

From equation (82) we can see that

$$Q_T^{cgp} = \frac{\beta - m}{2(n+\gamma)} + \frac{n(x_{cg}^{cgp} - Q_d)}{(n+\gamma)} = Q_T^{scg} + \frac{n(x_{cg}^{cgp} - Q_d)}{(n+\gamma)}$$
(Non-Arbitrage Mode) or
$$Q_T^{cgp} = \frac{\beta - m}{2(n+\gamma)} - \frac{\gamma Q_d}{(n+\gamma)} + \frac{n(x_{cg}^{cgp} - Q_d)}{(n+\gamma)} = Q_T^{scg} - \frac{\gamma Q_d}{(n+\gamma)} + \frac{n(x_{cg}^{cgp} - Q_d)}{(n+\gamma)}$$
(Arbitrage Mode)

If the equilibrium solution in the CGP model is non-arbitrage mode, binding or nonbinding, the total electricity supply to the retail electricity customers is always greater than the total electricity supply in the SCG model. If the equilibrium solution in the CGP model is the arbitrage mode then from expressions $Q_T^{cgp} > Q_T^{scg}$ if and only if $x_{cg}^{scg} > \frac{2n+\gamma}{2n}Q_d$. The cost of generation to the electric utility is less in the SCG model than in the CGP model. This is because the electric utility's optimal generation quantity is less in the CGP model than in the SCG model. From equations (26), (28) and (39) we have the following relationship between the electric utility's optimal generation quantity in the CGP model and the electric utility's optimal generation quantity in the SCG model

$$x_{cg}^{cgp} = x_{cg}^{scg} - (x_{cg}^{cgp} - \frac{Q_d}{2})\frac{\gamma}{n+\gamma}$$
(AM) or $x_{cg}^{cgp} = x_{cg}^{scg} - (x_{cg}^{cgp} - Q_d)\frac{\gamma}{n+\gamma}$ (NAM)

Since $\alpha S_d > Q_d$ by assumption and $x_{cg}^{cgp} \ge \alpha S_d$, $x_{cg}^{cgp} > Q_d$. Therefore $x_{cg}^{cgp} < x_{cg}^{scg}$ and and $C_u(x_{cg}^{cgp}) < C_u(x_{cg}^{scg})$.



If the equilibrium solution in the CGP model is binding for the cogeneration facility (i.e. $x_{cg}^{cgp} = \alpha S_d$) then the cost of cogeneration to the cogeneration facility remains the same in both the SCG and CGP models. If the equilibrium solution in the CGP model is nonbinding for the cogeneration facility (i.e. $x_{cg}^{cgp} \ge \alpha S_d$) then the cost of cogeneration to the cogeneration facility is higher in the CGP configuration than in the SCG configuration. Therefore if the parameters, { $\alpha, \beta, \gamma, m, n, c_1, c_2, S_d, Q_d$ }, are such that the CGP model's equilibrium solution is non-arbitrage and binding for the cogeneration facility, the total surplus of the CGP model will always greater than the total surplus of the SCG model. If the parameter set is such that the equilibrium solution of the CGP model is arbitrage and binding for the cogeneration facility then the total surplus of the CGP model will be greater than the total surplus of the SCG model if and only if $x_{cg}^{cgp} > ((2n + \gamma)Q_d)/2n$. If the parameter set is such that the CGP model's equilibrium solution in non-binding (arbitrage or non-arbitrage) for the cogeneration facility, the total surplus of the CGP model will be greater than the total surplus of the SCG model if and only if $x_{cg}^{cgp} > ((2n + \gamma)Q_d)/2n$. If the parameter set is such that the CGP model's equilibrium solution in non-binding (arbitrage or non-arbitrage) for the cogeneration facility, the total surplus of the CGP model will be greater than the total surplus of the SCG model under the following condition,

$$\nu(Q_T^{cgp}) - \nu(Q_T^{scg}) + C_u(x_u^{scg}) - C_u(x_u^{cgp}) < \bar{C}_{cg}(x_{cg}^{cgp}) - \bar{C}_{cg}(\alpha S_d)$$
(91)

If the cogeneration facility cogenerates more electricity than that required to satisfy the thermal host's heat demand, the total surplus of the CGP configuration will be greater than that of the SCG configuration is that the additional cost of cogeneration incurred by the thermal host in the CGP model is less than the increase in economic utility to the wholesale electricity customers in the CGP model and the cost reduction to the electric utility in the CGP model.



CHAPTER 3. ENVIRONMENTAL BENEFITS OF COGENERATION UNDER PURPA

3.1 Introduction and overview of emission control in electricity generation

In this chapter of the dissertation we study the generation planning problem and the associated environmental performance of a host utility and qualifying facility under a PURPA contract. The qualifying facility and host utility trade electricity due to the Public Utilities Regulatory Policies Act (PURPA) contract while the host utility is also regulated for its emissions of nitrogen oxides (NO_x) a seasonal cap and trade program. As in chapter 2, the interaction of the host utility and the qualifying facility as part of the PURPA contract is cast as a Stackleberg game with the host utility as the leader and the qualifying facility as the follower. The optimal generation plan for host utility and the qualifying facility are determined and the total NO_x emissions in the system associated with this optimal generation plan is calculated. The model is referred to as the Cogeneration under PURPA with emission control or CGPE model.

To evaluate the environmental performance of the CGPE model, the total NO_x emissions CGPE model is compared with the total emissions from the three benchmark models introduced in chapter 2. In all three benchmarks the electric utility's NO_x emissions will be regulated by a seasonal cap and trade program. The configurations of the benchmarks remain the same as in chapter 2 with the addition of the electric utility's NO_x emissions will be regulated by a seasonal cap and trade program. The symbol and definitions of the decision



variables and parameters used in this chapter are given in Table 7. The optimal value of the common decision variables in each configuration is denoted by adding a superscript to the variables symbol corresponding to the model name's acronym. For example, the optimal generation quantity of the host utility in the CGPE model is denoted by x_u^{cgpe} while the optimal generation quantity of the electric utility in the SCGE model is denoted by x_u^{scge} .

Symbol	Description		
Decision Varia	ables:		
x _u	Electricity generated by electric (host) utility (MBtu)		
x _{cg}	Electricity cogenerated by the qualifying facility (MBtu);		
y _{cg}	Heat energy cogenerated by the qualifying facility		
q_s	Electricity sold by the qualifying facility to the host utility (MBtu) (In CGPE model only)		
q_b	Electricity purchased by the qualifying facility from the host utility (MBtu)		
P _r	Electricity price paid by the retail electricity customers (\$/MBtu)		
Parameters:			
Q _d	Electricity demand of the thermal host (MBtu)		
S _d	Heat demand of the thermal host (MBtu)		
P _s	PURPA buyback price at which the qualifying facility sells electricity to the host utility as part of		
	the PURPA contract (\$/MBtu).		
α	Power to heat ratio of the cogeneration facility (constant)		
P _n	Price of an No_x allowance in the allowance market (\$/ton)		
e_u, e_{cg}, e_h	No _x emission rate of the electric utility, cogeneration facility and het production unit respectively		
	(lbs/MBtu)		
A _u	Annual No _x allowance allocation for the electric utility (tons)		

Table 7. Notations in chapter 3.



Emission control is a critical issue is electricity generation. It has been estimated that electricity generation is responsible for 62.6% of sulfur dioxide emissions, 21.2% of nitrogen oxides emissions and 40% of carbon dioxide emissions. These emissions are directly linked to air pollution and climate change issues. Hence in the 1990's the emissions from electricity generation units are regulated by many programs. The Clean Air Act was passed in 1990 and all electricity producers of capacity greater than 25MW are regulated for sulfur dioxide (SO₂) emissions and nitrogen oxides (NO_x) emission. The regulatory programs associated with the Clean Air Act was a federal cap and trade program for SO₂ that was phased in over a decade and a mandatory NO_x emission limit for coal based power plants enforced on generation units. Based on the success of the national cap and trade program in 1994, a regional cap and trade program for both sulfur oxides (SO_x) and nitrogen oxides (NO_x) was implemented in the South Coast Area Basin. This was followed by a seasonal NO_x trading program in 1999 that was implemented in North eastern United States to combat ozone formation. Though carbon dioxide emissions are not yet federally regulated, regional and state based efforts to bring about carbon dioxide (CO_2) trading programs are being implemented in Northeastern states and Western states of the country (Rose et al, 2006).

Hence since 1990, the generation of electricity is closely related to emission control and regulatory programs. In chapter 2 of the dissertation we evaluated the economic performance of PURPA. In this chapter we extend the basic model to include the emission regulatory program and evaluate the environmental performance of our main model – the CGPE model.



We specifically focus on the NO_x emissions. This is because the majority of cogeneration capacity is located in Texas, the Northeast and California totaling about 42% of the country's cogeneration capacity (USCHPA, 2001). In all the above mentioned states, the electric utilities have their have their NO_x emissions regulated by the CAIR program or the RECLAIM program. Being a part of the regulatory program and participating in the permits market will have a direct impact on their generation planning and hence their interaction with qualifying facilities. Therefore, it is critical to study the impact the NO_x regulatory program has on the relationship between the host utility and the qualifying facility.

3.1.1 Overview of NO_x emission regulatory programs

In this paper we focus on the regulation of NO_x emissions. We focus on the ozone season cap and trade program that have been in effect in the North eastern United States since 1999. In 1999 the Ozone Transport Commission's NO_x Budget Program (OTC) came into effect. It was transitioned into the larger NO_x Budget Trading Program (NBP) in 2004. The NBP covered a larger geographic region and had more stringent rules than the OTC program. The year 2008 was to be the last year of the NBP program with it transitioning into the Clean Air Interstate Rule's (CAIR) seasonal program was to come into effect in 2009. The CAIR ozone program expanded the geographic region to include mid-western states and Texas. In addition CAIR also included an annual NO_x and SO₂ programs. The NO_x Budget Trading Program (NBP) that s the was in effect since 2004 in 19 eastern states and was to be transitioned into the CAIR program cease to be in effect after the ozone season of 2008.



The models in this paper will be based on the seasonal CAIR ozone program for NO_x emissions. If and when the program or some modified version is implemented the program will still be a cap and trade program. The NO_x program is in effect from of May 1st to September 30th, the summer months, when ozone pollution is most prevalent. The participating states allocate allowances to the affected units along with a cap on the total emissions in the state during the annual ozone season of May 1st to September 30th. The affected units should retire one allowance for each ton of NO_x that they emit during the 5 month period. The EPA will oversee a regional allowance market where the affected units can trade allowances. The types of units that are regulated by the CAIR program include all fossil –fuel burning boilers serving generators of capacity greater than or equal to 25 MW that generate electricity for sale. The electric utility studied in this paper is regulated by the CAIR program include all the context of the service of the serv

The CAIR program provides exemption for units that qualify as cogeneration units which meet certain efficiency standards and sell no more than one third of their total electricity generation or 219, 000 MWh whichever is greater of electricity on an annual basis. This exemption for qualifying facilities and small power producers has been a part of all the emission control regulatory programs that have been in effect since the Clean Air Act. In our paper we study the scenario when the cogeneration facility is not part of any regulatory program.

However it should be noted that in September 2008, the CAIR program was taken to court and due to the outcome of these legal proceedings the program has been put on hold and the Environmental Protection Agency (EPA) requested to re-evaluate the program (EPA,



2008). However, several generating utility companies have already installed emission monitoring devices and purchased permits in preparation for the CAIR regulatory program. It is believed that to prevent losses in profit and health benefits and prevent electricity generation costs from being increased some form of seasonal NO_x program will be put into effect in 2009 while the EPA pursues other options. Proposals include continuation of the NO_x Budget Trading Program (NBP) in 2009 (Mathias, 2008). Hence the models we study in this paper will remain appropriate.

3.2 Literature Review

The effect of the NO_x trading program on the generation planning of an electric utility has also been previously studied. The OTC Budget program has been studied with reference to market power and leader follower behavior in Chen and Hobbs (2005) and Chen et al (2006). In both papers, the permits market was modeled as a oligopolistic market with a few major firms who's permits output into the market determines the price of the permits. They do not explicitly model cogeneration facilities or account for power purchases by host utilities due to PURPA. The environmental implications of cogeneration have been studied extensively. While most of the studies have focused on solution to the economic dispatch problem of cogeneration units with emission constraints (Venkatesh et al, 2003; Tsay et al, 2001) a few have focused on the generation planning aspect of cogeneration units. In Wu and Rosen (1999), they develop an energy equilibrium model to identify the environmental benefits of cogeneration in reference to distributed generation. However the study is based on generation units in Canada and do not address PURPA or the impact of PURPA on the emissions from



the host utility. Similarly, in Rong and Lahdelma (2007), the authors consider the effect of CO_2 emissions trading on the production plan of a cogeneration facility. Their study is a multi-period model that accounts from uncertainty in heat demand, electricity price and permit price. Once again the study was based on generation systems outside of the United States and does not address the impact of any regulations other than CO_2 emission regulations on the production plan of the cogeneration facility. In this part of the thesis we aim to study how the presence of a PURPA contract impacts the behavior of an electric utility in terms of NO_x emissions. We also hope to compare the environmental performance of the cogeneration under PURPA model with the three benchmarks that we have developed in chapter 2.

3.3 Modeling Assumptions

In addition to the modeling assumptions listed in chapter 2, due the inclusion of emission control and participation of the electric utility in the permits market there are certain additional assumptions.

A9: The electric utility is the price taker in the permits market. This assumption is justified, since Chen and Hobbs (2005) considered the Pennsylvania-New Jersey-Maryland (PJM) power market and the OTC No_x Budget Program's permits market covering the 12 states of Connecticut, Delaware, Maryland, Massachusetts, New Hampshire, New Jersey, Pennsylvania, Vermont, New York, Rhode Island, Maine, Northern Counties of Virginia and District of Columbia (Overview of OTC Budget Program, 2008).



A10: It is also assumed that the electric utility will sell all excess permits in the permits market, i.e. permits are not banked for future use. This assumption is justified since in the NBP program, banked permits were devalued as time progressed. In effect after a period of time one permit will not cover one ton of pollutant. This was referred to as Progressive Flow Control (PFC).

3.4 Cogeneration under PURPA with emission control (CGPE) model

In the CGPE model the interaction between the qualifying facility and the host utility remain the same as in chapter 2. However, in the CGPE model, the electric utility's No_x emissions are regulated as part of a cap and trade program. Hence the host utility is a participant in the No_x permits market. The system configuration of the CGPE model is given in Figure 6.



Figure 6. Cogeneration under PURPA configuration with emission control (CGPE)

model



The generation planning problem of the qualifying facility, its optimal solution and its reaction function remain the same as in chapter 2. However, the host utility's generation planning problem is modified to include its participation in the No_x regulatory program.

3.4.1 Host utility's generation planning problem with emission control

The profit function π_{eu}^{cgpe} , of the host utility consists of five terms – revenue from retail electricity sales, cost of generation, cost of purchasing cogenerated electricity from the qualifying facility as part of the PURPA contract, the revenue from selling electricity to the cogeneration facility at the electricity price P_r in a bilateral transaction and revenue/cost from selling or purchasing permits in the No_x permits market. The profit of the host utility is a function of the reactions of the qualifying facility. Similar to chapter 2, the host utility has two possible generation planning problems to account for the two different reactions of the qualifying facility.

Non-Arbitrage Mode ($P_r \ge P_s$): In this case the reaction of the qualifying facility from equation (18) is substituted in the profit function π_{eu}^{cgpe} of the host utility. The profit maximization problem (P6) of the host utility is modified as follows,

$$\underbrace{Max}_{x_{u}} \pi_{eu}^{cgpe} = \beta (x_{u} + x_{cg} - Q_{d}) - \gamma (x_{u} + x_{cg} - Q_{d})^{2} - C_{u}(x_{u}) - P_{s}(x_{cg} - Q_{d}) - \gamma (x_{u} + x_{cg} - Q_{d})^{2} - C_{u}(x_{u}) - P_{s}(x_{cg} - Q_{d}) - \gamma (x_{u} + x_{cg} - Q_{d})^{2} - C_{u}(x_{u}) - P_{s}(x_{cg} - Q_{d}) - \gamma (x_{u} + x_{cg} - Q_{d})^{2} - C_{u}(x_{u}) - P_{s}(x_{cg} - Q_{d}) - \gamma (x_{u} + x_{cg} - Q_{d})^{2} - C_{u}(x_{u}) - P_{s}(x_{cg} - Q_{d}) - \gamma (x_{u} + x_{cg} - Q_{d})^{2} - C_{u}(x_{u}) - P_{s}(x_{cg} - Q_{d}) - \gamma (x_{u} + x_{cg} - Q_{d})^{2} - C_{u}(x_{u}) - P_{s}(x_{cg} - Q_{d}) - \gamma (x_{u} + x_{cg} - Q_{d})^{2} - C_{u}(x_{u}) - P_{s}(x_{cg} - Q_{d}) - \gamma (x_{u} + x_{cg} - Q_{d})^{2} - C_{u}(x_{u}) - P_{s}(x_{cg} - Q_{d}) - \gamma (x_{u} + x_{cg} - Q_{d})^{2} - C_{u}(x_{u}) - P_{s}(x_{cg} - Q_{d}) - \gamma (x_{u} + x_{cg} - Q_{d})^{2} - C_{u}(x_{u}) - P_{s}(x_{cg} - Q_{d}) - \gamma (x_{u} + x_{cg} - Q_{d})^{2} - C_{u}(x_{u}) - P_{s}(x_{cg} - Q_{d}) - \gamma (x_{u} + x_{cg} - Q_{d})^{2} - C_{u}(x_{u}) - P_{s}(x_{cg} - Q_{d}) - \gamma (x_{u} + x_{cg} - Q_{d})^{2} - C_{u}(x_{u}) - P_{s}(x_{cg} - Q_{d}) - \gamma (x_{u} + x_{cg} - Q_{d})^{2} - C_{u}(x_{u}) - P_{s}(x_{cg} - Q_{d}) - \gamma (x_{u} + x_{cg} - Q_{d})^{2} - C_{u}(x_{u}) - P_{s}(x_{cg} - Q_{d}) - \gamma (x_{u} + x_{cg} - Q_{d})^{2} - C_{u}(x_{u}) - P_{s}(x_{cg} - Q_{d})^{2} - C_{u}(x_{u}) - C_{$$

$$P_n(e_u x_u - A_u) \quad \text{s.t} \quad x_u \le \frac{1}{\gamma} [\beta - P_s - \gamma (x_{cg} - Q_d)]$$
(94)

The problem P6 of the host utility is a non-linear problem with linear constraint. The corresponding Lagrangian function is

$$L_{eu}^{cgpe} = \beta \left(x_u + x_{cg} - Q_d \right) - \gamma \left(x_u + x_{cg} - Q_d \right)^2 - C_u (x_u) - P_s (x_{cg} - Q_d) - P_n (e_u x_u - A_u) - \lambda_1^{cgpe} \left(x_u - \frac{1}{\gamma} [\beta - P_s - \gamma \left(x_{cg} - Q_d \right) \right)$$
(95)



Therefore the optimal x_u can be determined from the first order necessary and sufficient conditions as follows

FONC:
$$\frac{\partial \pi_{eu}^{cgpe}}{\partial x_u} = \beta - 2\gamma (x_u + x_{cg} - Q_d) - m - 2nx_u - P_n e_u = 0$$
(96)

$$x_u^{cgpe_NB_NAM} = \frac{\beta - m - P_n e_u - 2\gamma(x_{cg} - Q_d)}{2(\gamma + n)}$$
(97)

Equation (97) is the non-binding solution of problem P6. The optimal solution if the constraint is binding is $x_u^{cgpe_B_NAM} = \frac{1}{\gamma} \left[\beta - P_s - \gamma (x_{cg} - Q_d)\right]$, when $\lambda_1^{cgpe} \ge 0$. When the optimal solution is binding, $\lambda_1^{cgpe} = \frac{2n\gamma x_{cg} + 2P_s(n+\gamma) - 2n\beta - \gamma (m+\beta + e_u P_n) - \gamma Q_d 2n}{\gamma}$. This leads to a condition on the electricity demand Q_d , of the thermal host based on which the optimal generation quantity of the electric utility x_u , is such that $P_r \ge P_s$.

$$\frac{2n\gamma x_{cg} + 2P_s(n+\gamma) - 2n\beta - \gamma(m+\beta + e_u P_n)}{2n\gamma} < Q_d \tag{98}$$

Therefore the optimal solution to the host utility's generation planning problem in the Non-Arbitrage mode is

$$x_{u}^{cgpe_nam} = \begin{cases} \frac{\beta - m - P_{n}e_{u} - 2\gamma(x_{cg} - Q_{d})}{2(\gamma + n)}, \text{ when } Q_{d} > \frac{2n\gamma x_{cg} - 2n\beta - (m + \beta + e_{u}P_{n})\gamma + 2P_{s}(n + \gamma)}{2n\gamma} \\ \frac{1}{\gamma} [\beta - P_{s} - \gamma(x_{cg} - Q_{d})], \text{ otherwise} \end{cases}$$
(99)

Case (b): Arbitrage Mode ($P_r < P_s$ *)* In this case the profit maximization problem (P7) of the host utility is modified as follows,

$$\underbrace{Max}_{x_{u}} \pi_{eu}^{cgpe} = \beta (x_{u} + x_{cg} - Q_{d}) - \gamma (x_{u} + x_{cg} - Q_{d})^{2} - C_{u}(x_{u}) - P_{s}x_{cg} + Q_{d}\beta - \gamma Q_{d}(x_{u} + x_{cg} - Q_{d}) - P_{n} (e_{u}x_{u} - N_{u}) \text{ s.t } x_{u} > \frac{1}{\gamma} [\beta - P_{s} - \gamma (x_{cg} - Q_{d})]$$
(100)



The problem P7 is a non-linear problem with linear constraints and the optimal x_u can be determined by using the FONC to be

$$x_u^{cgpe_NB_AM} = \frac{\beta - m - P_n e_u - 2\gamma x_{cg} + \gamma Q_d}{2(\gamma + n)}$$
(101)

Since the constraint in P7 is a strict inequality, the optimal non-binding solution in equation (101) should be strictly less than the upper bound specified in equation (101). The condition that ensures this is $\lambda_1^{cgpe} = \frac{2n\gamma x_{cg} + 2P_s(n+\gamma) - 2n\beta - \gamma(m+\beta + e_u P_n) - \gamma(\gamma + 2n)Q_d}{\gamma} \ge 0$. Similar to the non-arbitrage case we can convert the condition to a limit on the electricity demand Q_d of the thermal host. Therefore the optimal generation optimal generation quantity x_u is such that P_r

$$< P_s$$
 is

$$x_{u}^{cgpe_am} = \frac{\beta - m - P_{n}e_{u} - 2\gamma(x_{cg} - Q_{d})}{2(\gamma + n)}, \text{ when } Q_{d} < \frac{2n\gamma x_{cg} - 2n\beta - (m + \beta + e_{u}P_{n})\gamma + 2P_{s}(n + \gamma)}{\gamma(\gamma + 2n)}$$
(102)
Let CN5 =
$$\frac{2n\gamma x_{cg} - 2n\beta - (m + \beta + e_{u}P_{n})\gamma + 2P_{s}(n + \gamma)}{2n\gamma} \text{ and}$$
CN6 =
$$\frac{(2n\gamma x_{cg} - 2n\beta - (m + \beta + e_{u}P_{n})\gamma + 2(n + \gamma)P_{s})}{\gamma(\gamma + 2n)}.$$
(103)

It can be seen that CN6 > CN5.

3.4.2 Equilibrium solution of CGPE model

In this section we define the Stackelberg equilibrium for the game between the electric utility and cogeneration facility when the cogeneration facility is an Independent Power Producer. The equilibrium solution is one from which no player has an incentive to change from (Gibbons 1992).



Definition of Stackelberg Equilibrium: The equilibrium solution to the Stackelberg game in the CGP configuration is defined as the set of decisions, $\{x_u^*, P_r^*, x_{cg}^*, y_{cg}^*, q_s^*, q_b^*\}$ that satisfy the following conditions

- The generation quantity x_u and the associated electricity price P_r maximizes the host utility's profit and results in the optimal compliance with the NO_x regulatory program when she has a valid PURPA contract with a qualifying facility as part of which she trades electricity.
- The cogeneration output of electricity x_{cg} and heat y_{cg} , electricity sold to the host utility q_s and electricity purchased from the host utility q_b for a given electricity price P_r maximizes the qualifying facility's profit and satisfies the electricity and heat demand of the thermal host

Similar to the CGP model in chapter 2, the CGPE model has six possible solutions of which for a given set of parameters { α , β , γ , m, n, S_d , Q_d , c_1 , c_2 } we have mutually exclusive conditions those results in an unique equilibrium. A summary of the equilibrium solutions to the CGPE model is given in Table 8.



Case	P_r^{cgpe}	x_u^{cgpe}	x_{cg}^{cgpe}			
Non Binding Case						
<i>Q_d</i> < CN2	$P_r < P_s$	$\frac{\beta - m - P_n e_u - 2\gamma x_{cg} + \gamma Q_d}{2(\gamma + n)}$	$\frac{P_s - c_1}{2c_2}$			
$CN2 \le Q_d <$	$P_r = P_s$	$\left[\beta-P_s-\gamma(x_{cg}-Q_d)\right]$	$\frac{P_s - c_1}{2c}$			
CN1		γ	202			
$Q_d > CN1$	$P_r > P_s$	$\frac{\beta - m - P_n e_u - 2\gamma(x_{cg} - Q_d)}{2(\gamma + n)}$	$\frac{P_s - c_1}{2c_2}$			
Binding Case						
$Q_d < CN2$	$P_r < P_s$	$\frac{\beta - m - P_n e_u - 2\gamma x_{cg} + \gamma Q_d}{2(\gamma + n)}$	αS_d			
$CN2 \le Q_d <$	$P_r = P_s$	$\left[\beta - P_s - \gamma (x_{cg} - Q_d)\right]$	αS_d			
CN1		γ				
$Q_d > CN1$	$P_r > P_s$	$\frac{\beta - m - P_n e_u - 2\gamma(x_{cg} - Q_d)}{2(\gamma + n)}$	αS _d			
Constants	$CN1 = \frac{2n\gamma x}{r}$	$\frac{1}{2n\gamma} \frac{1}{2n\gamma} (m+\beta+e_u P_n)\gamma+2P_s(n+\gamma)}{2n\gamma}; \text{CN2} = \frac{(2n\gamma x_{cg}-2n\beta-(n+\beta))}{2n\gamma}$	$\frac{n+\beta+e_uP_n)\gamma+2(n+\gamma)P_S}{\gamma(\gamma+2n)};$			
	$c_2 = \frac{c\alpha^2 + f\alpha}{\alpha^2}$	$\frac{\alpha+e}{\alpha}; \ c_1 = \frac{b\alpha+d}{\alpha};$				

Table 8: Equilibrium solution(s) of the CGP configuration Stackelberg game

To determine the environmental performance of the CGPE configuration we compute the total No_x emissions. The No_x emissions from the electric utility are straight forward to estimate. However for the cogeneration facility, the No_x emissions estimate should account for the fact that the use of the cogeneration technology displaces the emissions that will be generated if the thermal host's electricity and heat demand are satisfied by separate heat and electricity generation. Hence we estimate the net No_x emissions from the cogeneration



facility. This calculation is based on the CHP emission calculator developed by the EPA (EPA, 2008).

Therefore the net emissions is calculated as follows

Net Emissions = Cogeneration facility emissions – Displaced Thermal- Displaced Electricity Hence the net emissions from the cogeneration facility in the CGPE configuration

$$NE_{cgpe} = e_{cg} x_{cg}^{cgpe} - e_u Q_d - e_h S_d \tag{104}$$

It should be noted though that if at the equilibrium solution the qualifying facility operates in the Arbitrage mode then the emissions displaced due to the generation of thermal host's electricity demand should not be included in the calculation of the net emissions from the qualifying facility. This is because even though the qualifying facility generates the quantity (thermal host's electricity demand), it sells the quantity to the host utility and purchases the same quantity from the host utility.

Therefore, the total No_x emissions in the CGPE configuration is utility and the cogeneration facility

$$TE_{cgpe} = e_u x_u^{cgpe} + NE_{cgpe} = e_u x_u^{cgpe} + e_{cg} x_{cg}^{cgpe} - e_u Q_d - e_h S_d \text{ (Non-Arbitrage)}$$

or $TE_{cgpe} = e_u x_u^{cgpe} + NE_{cgpe} = e_u x_u^{cgpe} + e_{cg} x_{cg}^{cgpe} - e_h S_d \text{ (Arbitrage)}$ (105)
where x_u^{cgpe} and x_{cg}^{cgpe} are the electricity generated by the electric utility and cogenerated by

the cogeneration facility at equilibrium.



3.5 Cogeneration facility as an Independent Power Producer with emission control (IPPE) model

In the IPPE model too the cogeneration facility's generation planning problem, solution and reaction to the electric utility's decisions remain the same as in the case of chapter 2. The electric utility's generation planning problem and the equilibrium solution of the Stackelberg game in the IPPE model are presented below. The configuration of the IPPE model is shown in Figure 7.



Figure 7. Cogeneration facility as an Independent Power Producer configuration with emission control (IPPE) model



3.5.1 Electric utility's generation planning problem with emission control

Therefore, the residual demand, $R_u(P_r) = Q_T(P_r) - S_{cg}(P_r)$ is the generation quantity of the electric utility. The electric utility being the Stackelberg leader is able to choose the electricity price that maximizes its profit. The reaction function of the cogeneration facility determines the residual demand function of the electric utility. The reaction function however is discontinuous with the discontinuity occurring when $P_r \leq c_1 + 2c_2\alpha S_d$. If $P_r > c_1 + 2c_2\alpha S_d$, the reaction of the cogeneration facility is a function of the electricity price. If $P_r \leq c_1 + 2c_2\alpha S_d$, the cogeneration facility's reaction is to supply fixed quantity of electricity, $\alpha S_d - Q_d$ to the retail electricity customers. This leads to two cases – case (a) when $x_{cg}^{ippe} = \alpha S_d$ and case (b) when $x_{cg}^{ippe} = \frac{P_r - c_1}{2c_2} > \alpha S_d$

Case (a) $x_{cg}^{ippe}(P_r) = \alpha S_d$

Similar to chapter 2 we obtain the residual demand faced by the electricity based on the reaction of the cogeneration facility. The generation planning problem of the electric utility when $x_{cg}^{ippe}(P_r) = \alpha S_d$, is the profit maximization problem (P13) given below,

$$\begin{aligned}
& \underset{x_{u}}{\overset{Max}{x_{u}}} \pi_{eu}^{ipp} = P_{r}(x_{u})x_{u} - C_{u}(x_{u}) - P_{n}(e_{u}x_{u} - A_{u}) \\
& \text{s.t } x_{u} \ge 1/\gamma \left(\beta - \gamma(\alpha S_{d} - Q_{d}) - c_{1} - 2c_{2}\alpha S_{d}\right)
\end{aligned} \tag{106}$$

where $P_r(x_u) = \beta - \gamma(x_u + x_u - Q_d)$ is the residual inverse demand function faced by the electric utility and $C_u(x_u) = l + mx_u + nx_u^2$ is the cost of generation to the electric utility. Problem P13 is a non-linear constrained optimization problem. The optimal generation quantity for the electric utility can be obtained using the concept of active and inactive



constraints. The optimal unconstrained solution to problem P13 is obtained from the FONC of P13.

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FONC:

$$\frac{\partial \pi_{eu}^{ipp}}{\partial x_u} = -m - 2nx_u - \frac{2\gamma c_2 x_u}{\gamma + 2c_2} + \frac{2\beta c_2 - \gamma (-c_1 + 2c_2(-Q_d + x_u))}{\gamma + 2c_2} = 0$$

Therefore,

$$x_u^{ippe-B-NB} = \frac{\beta - m - P_n e_u - 2\gamma(\alpha S_d - Q_d)}{2(n+\gamma)} \tag{107}$$

where $x_u^{ippe-B-NB}$ is the unconstrained optimal solution of the optimization problem P13. The constrained solution to P13 is $x_u^{ippe-B-B} = \frac{\beta - \gamma(\alpha S_d - Q_d) - c_1 - 2c_2 \alpha S_d}{\gamma}$. The condition under which the optimal solution will be the constrained solution or the unconstrained solution can be obtained using the KKT conditions associated with problem P13. The Lagrangian function associated with P13 is

$$L_{eu}^{ipp} = P_r(x_u)x_u - C_u(x_u) - P_n(e_u x_u - A_u) + \mu_1^{ippe}(x_u - \frac{1}{\gamma}(\beta - \gamma(\alpha S_d - Q_d) - c_1 - 2c_2\alpha S_d))$$
(108)

$$\frac{\partial L_{eu}^{ipp}}{\partial x_u} = -m - 2nx_u - \frac{2\gamma c_2 x_u}{\gamma + 2c_2} + \frac{2\beta c_2 - \gamma (-c_1 + 2c_2(-Q_d + x_u))}{\gamma + 2c_2} + \mu_1^{ippe} = 0$$
(109)

$$\frac{\partial \pi_{eu}^{ipp}}{\partial x_u} = x_u - \frac{1}{\gamma} (\beta - \gamma (\alpha S_d - Q_d) - c_1 - 2c_2 \alpha S_d) = 0$$
(110)

$$\mu_{1}^{ippe}\left(x_{u} - \frac{1}{\gamma}(\beta - \gamma(\alpha S_{d} - Q_{d}) - c_{1} - 2c_{2}\alpha S_{d})\right) = 0; \ \mu_{1}^{ippe} \ge 0$$
(111)

From equation (111) we have that $x_u^{ippe-B-B} = \frac{\beta - \gamma(\alpha S_d - Q_d) - c_1 - 2c_2 \alpha S_d}{\gamma}$ when $\mu_1^{ippe} \ge 0$. We determine μ_1^{ippe} by substituting $x_u^{ippe-B-B}$ in equation (109) and solving for μ_1^{ippe} .

$$m\gamma + \beta(2n+\gamma) - 2(n+\gamma)c_1 + \gamma e_u P_n + (2n\gamma + \gamma^2)Q_d - (2n\gamma + \gamma^2 + 4(n+\gamma)c_2)\alpha S_d \ge 0 \quad (112)$$



Since the optimal solution to P13 is $x_u^{ippe-B-B}$ if $\mu_1^{ippe} \ge 0$, the optimal solution to P13 will be $x_u^{ippe-B-NB}$, if $\mu_1^{ippe} < 0$. $m\gamma + \beta(2n+\gamma) - 2(n+\gamma)c_1 + \gamma e_u P_n + (2n\gamma + \gamma^2)Q_d - (2n\gamma + \gamma^2 + 4(n+\gamma)c_2)\alpha S_d < 0$ (113) Let

$$CN7 = m\gamma + \beta(2n+\gamma) - 2(n+\gamma)c_1 + \gamma e_u P_n + (2n\gamma + \gamma^2)Q_d - (2n\gamma + \gamma^2 + 4(n+\gamma)c_2)\alpha S_d (114)$$

Case (b): $x_{cg}^{ipp}(P_r) = \frac{P_r - c_1}{2c_2}$

When $x_{cg}^{ipp}(P_r) = \frac{P_r - c_1}{2c_2}$ is the reaction of the cogeneration facility we obtain the residual

inverse demand of the electric utility as

$$P_r(x_u) = \frac{2c_2\beta - \gamma(2c_2(x_u - Q_d) - c_1)}{2c_2 + \gamma}$$
(115)

The generation planning problem of the electric utility when $x_{cg}^{ipp}(P_r) = \frac{P_r - c_1}{2c_2}$ is the profit

maximization (P14) problem given below,

$$\frac{Max}{x_{u}}\pi_{eu}^{ipp} = \left(\frac{2c_{2}\beta - \gamma(2c_{2}(x_{u} - Q_{d}) - c_{1})}{2c_{2} + \gamma}\right)x_{u} - l - mx_{u} - nx_{u}^{2} - P_{n}(e_{u}x_{u} - A_{u})$$
s.t $x_{u} < 1/\gamma \left(\beta - \gamma(\alpha S_{d} - Q_{d}) - c_{1} - 2c_{2}\alpha S_{d}\right)$
(116)

The optimal generation quantity of the electric utility in this situation is

$$x_{u}^{ippe_NB} = \frac{2c_{2}(\beta + \gamma Q_{d} - m - P_{n}e_{u}) + \gamma(c_{1} - m - P_{n}e_{u}))}{2(n\gamma + 2(n+\gamma)c_{2})}$$
(117)

where $x_u^{ippe_NB}$ is the unconstrained solution of the optimization problem P14. The condition under which the $x_u^{ippe_NB}$ will be a feasible solution to P14 is

$$\frac{2c_{2}(\beta + \gamma Q_{d} - m - P_{n}e_{u}) + \gamma(c_{1} - m - P_{n}e_{u}))}{2(n\gamma + 2(n+\gamma)c_{2})} < 1/\gamma \left(\beta - \gamma(\alpha S_{d} - Q_{d}) - c_{1} - 2c_{2}\alpha S_{d}\right)$$



i.e.
$$2(n\gamma^2 + 4n\gamma c_2 + 2\gamma^2 c_2 + 4nc_2^2 + 4\gamma c_2^2)\alpha S_d - m(\gamma^2 + 2\gamma c_2) - 2\beta(n\gamma + 2nc_2 + \gamma c_2) - (\gamma^2 + 2\gamma c_2 euPn + c_{12}n\gamma + \gamma^2 + 4nc_2 + 4\gamma c_2 - 2n\gamma^2 + 2n\gamma c_2 + 2\gamma^2 c_2 Qd < 0$$
 (118)

Let

$$CN8 = 2(n\gamma^{2} + 4n\gamma c_{2} + 2\gamma^{2}c_{2} + 4nc_{2}^{2} + 4\gamma c_{2}^{2})\alpha S_{d} - m(\gamma^{2} + 2\gamma c_{2}) - 2\beta(n\gamma + 2nc_{2} + \gamma c_{2}) - (\gamma^{2} + 2\gamma c_{2}c_{2})m + c^{2}m\gamma c_{2} + \gamma c_{2} + 4\gamma c_{2}^{2} - 2n\gamma c_{2}^{2} + 2\gamma c_{2}^{2}Qd$$
(119)

3.5.2 Equilibrium solution of IPPE model

A summary of the equilibrium solutions of the IPPE model is provided below in Table 9.

Condition	x_u^{ipp}	x_{cg}^{ipp}				
Non-Binding;	$2c_2(\beta + \gamma Q_d - m - P_n e_u) + \gamma (c_1 - m - P_n e_u))$	$P_r - c_1$				
CN8 < 0	$2(n\gamma+2(n+\gamma)c_2)$	$2c_2$				
Binding with	$\beta - m - P_n e_u - 2\gamma(\alpha S_d - Q_d)$	αS_d				
$P_r = c_1 + 2c_2\alpha S_d$	$2(n+\gamma)$					
;						
$CN7 \ge 0$						
Binding with	$(\beta - \gamma(\alpha S_d - Q_d) - c_1 - 2c_2\alpha S_d)$	αS_d				
$P_r < c_1 + 2c_2\alpha S_d;$	γ					
CN7 < 0						
Constants: $c_2 = \frac{c\alpha^2 + f\alpha + e}{\alpha^2}; c_1 = \frac{b\alpha + d}{\alpha};$						
$CN7 = m\gamma + \beta(2n+\gamma) - 2(n+\gamma)c_1 + \gamma e_u P_n + (2n\gamma + \gamma^2)Q_d - (2n\gamma + \gamma^2 + 4(n+\gamma)c_2)\alpha S_d;$						
$CN8 = 2(n\gamma^{2} + 4n\gamma c_{2} + 2\gamma^{2}c_{2} + 4nc_{2}^{2} + 4\gamma c_{2}^{2})\alpha S_{d} - m(\gamma^{2} + 2\gamma c_{2}) - 2\beta(n\gamma + 2nc_{2} + \gamma c_{2}) - (\gamma^{2} + 2\gamma c_{2}) - (\gamma$						
$2\gamma c2euPn+c12n\gamma+\gamma2+4nc2+4\gamma c2-2n\gamma2+2n\gamma c2+2\gamma2c2Qd;$						

Table 9. Equilibirum solution(s) of the IPPE configuration Stackleberg game

Hence the net emissions from the cogeneration facility in the CGPE configuration

$$NE_{ippe} = e_{cg} x_{cg}^{ippe} - e_u Q_d - e_h S_d$$



Therefore, the total No_x emissions in the IPPE configuration is utility and the cogeneration facility

$$TE_{ippe} = e_u x_u^{ippe} + NE_{cgpe} = e_u x_u^{ippe} + e_{cg} x_{cg}^{ippe} - e_u Q_d - e_h S_d$$
(120)

where x_u^{ippe} and x_{cg}^{ippe} are the electricity generated by the electric utility and cogenerated by the cogeneration facility at equilibrium.

3.6 Heat production without cogeneration with emission control (HPE) model

In the HPE model has the same energy generation system configuration as the HP model. The HPE model differs from the HP model in the electric utility's participation in the No_x market and its No_x emissions being regulated. The energy generation system configuration is shown in Figure 8.



Figure 8. Heat production without cogeneration configuration with emission control

(HPE) model



The total cost, C_T^{hpe} of satisfying the heat and electricity demand of the thermal host remains the same as in chapter 2,

$$C_T^{hp} = C_h(S_d) + P_r Q_d = i + jS_d + kS_d^2 + P_r Q_d.$$
(121)

where $C_h(S_d) = i + jS_d + kS_d^2$ is the cost of generating heat S_d by a heat production unit and P_r is electricity price at which the electric utility sells electricity to the thermal host in an independent transaction. The electric utility's generation planning problem is the profit maximization problem given below

$$\frac{Max}{x_u} \pi_{eu}^{hpe} = P_r(Q_T^{hpe})Q_T^{hpe} - C_u(x_u) + P_r(Q_T^{hpe})Q_d) - P_n(e_u x_u - A_u)$$
(122)

with $Q_T^{hpe} = x_u - Q_d$, the electricity supplied by the electric utility to the retail electricity customers and $P_r(Q_T^{hp}) = \beta - \gamma(Q_T^{hpe})$ the inverse demand function of the electric utility. $C(x_u) = l + mx_u + nx_u^2$ is the electricity generation cost to the electric utility. The profit maximization problem is a non-linear problem and the optimal generation quantity can be obtained from the first order necessary conditions as follows

FONC:
$$\frac{\partial \pi_{eu}^{hpe}}{\partial x_u} = \beta - 2\gamma x_u - m - 2nx_u + \gamma Q_d - P_n e_u = 0$$
 (123)

$$x_u^{hpe} = \frac{\beta - m - P_n e_u + \gamma Q_d}{2(n+\gamma)} \tag{124}$$

Since there is no cogeneration facility in this configuration the total No_x emissions in the HPE model is the emissions of the electric utility and the emissions form the boiler.

$$TE_{hpe} = e_u x_u^{hpe} + e_h S_d \tag{125}$$

where x_u^{hpe} is the electricity generated by the electric utility and S_d is the useful heat generated by the heat production unit.



3.7 Self-Generation with cogeneration and emission control (SGCE) model

In the SCGE model, the thermal host's heat demand and electricity demand is satisfied by a cogeneration facility that does not have interact with the electric utility or the retail electricity customers. The energy generation system configuration for the SGCE is shown in Figure 9.



Figure 9. Self generation with cogeneration and emission control (SCGE) model

The total cost, C_T^{scge} of satisfying the heat and electricity demand of the thermal host is given below

$$C_T^{scge} = \bar{C}_{cg}(\alpha S_d) = a + c_1 \alpha S_d + c_2 \alpha^2 S_d^{-2}.$$
 (126)

 $\bar{C}_{cq}(\alpha S_d)$ = Cost of cogenerating αS_d of electricity and S_d of heat.

The electric utility's generation planning problem is given below

$$\frac{Max}{x_u} \ \pi_{eu}^{scge} = P_r(x_u) x_u - C(x_u) - P_n(e_u x_u - A_u)$$
(127)



where $P_r(x_u) = \beta - \gamma x_u$ is the inverse demand function of the electric utility and $C_u(x_u) = l + mx_u + nx_u^2$ is the electricity generation cost to the electric utility. The optimal generation quantity for the electric utility is determined from the FONC as

$$x_u^{scge} = \frac{\beta - m - P_n e_u}{2(n+\gamma)} \tag{128}$$

The net emissions from the cogeneration facility in the SCG model is

$$NE_{scge} = e_{cg}\alpha S_d - e_u Q_d - e_h S_d \tag{129}$$

Therefore, the total No_x emissions in the CGPE configuration is utility and the cogeneration facility

$$TE_{scge} = e_u x_u^{scge} + NE_{scge} = e_u x_u^{cgpe} + e_{cg} \alpha S_d - e_u Q_d - e_h S_d$$
(130)
where x_u^{scge} is the electricity generated by the electric utility

3.8 Environmental performance of cogeneration under PURPA

w.r.t. the benchmarks

In this section of the paper compare the environmental performance of the cogeneration under PURPA configuration with the three benchmark configurations. Though environmental performance can be measured in many ways, we focus on the total NO_x emissions associated with optimal generation plan in each configuration results in.

The total NO_x emissions in the configurations studied in the paper is the sum of the emissions from the electric utility due to electricity generation and the net emissions from the cogeneration facility. The accounting of emissions from a cogeneration facility is not as straight forward as the calculation of emissions from an electric utility. This is due to the



generation of two useful energy products due to the cogeneration technology. The EPA's Combined Heat and Power Production (CHP) partnership program have developed a simple CHP emissions calculator. CHP is another name for cogeneration. In the paper we use the formula used in the EPA's CHP emissions calculator to account for the emissions from the cogeneration facility.

3.8.1 Comparison of IPPE model and CGPE model

In the comparison between the IPPE configuration model and the CGPE configuration model.

Case (a) Binding solution with $P_r^{ipp} < c_1 + 2c_2\alpha S_d$

The total NO_x emissions in the system in the IPPE model is

$$TE_{ippe} = e_u \frac{\beta - m - P_n e_u - \gamma \alpha S_d + \gamma Q_d}{2(\gamma + n)} + e_{cg} \alpha S_d - e_u Q_d - e_h S_d$$
(131)

If the equilibrium solution of the CGPE model is the arbitrage-binding solution, the total NO_x emissions generated in the system is

$$TE_{cgpe} = e_u \frac{\beta - m - P_n e_u - 2\gamma \alpha S_d + \gamma Q_d}{2(\gamma + n)} + e_{cg} \alpha S_d - e_h S_d$$
(132)

In equation (132) we do not consider the displaced emissions associated with the generation of the electricity demand of the thermal host by the electric utility. This is because, in the arbitrage solution (both binding and non-binding) of the CGPE model, the qualifying facility sells all its cogenerated electricity to the host utility and purchases electricity from the host utility to satisfy the thermal host's electricity demand. By comparing the total NO_x emissions expression in equations (131) and (132) we see that the only difference between the total NOx emissions is the based on the generation quantity of the host utility and the displaced



emissions due to the electricity demand of the thermal host being satisfied by the

cogeneration facility.

$$e_u \frac{\beta - m - P_n e_u - 2\gamma \alpha S_d + \gamma Q_d}{2(\gamma + n)} < e_u \frac{\beta - m - P_n e_u - \gamma \alpha S_d + \gamma Q_d}{2(\gamma + n)} - e_u Q_d$$

Based on the above inequality we identify the condition under which the emissions from the CGPE model will be lower than the emissions from the IPPE model to be

$$TE_{cgpe} < TE_{ippe}$$
 if and only if $\alpha S_d > \frac{2(\gamma+n)}{\gamma} Q_d$ (133)

If the equilibrium solution of the CGPE model is the non-arbitrage-binding solution,

$$TE_{cgpe} = e_u \frac{\beta - m - P_n e_u - 2\gamma \alpha S_d + 2\gamma Q_d}{2(\gamma + n)} + e_{cg} \alpha S_d - e_u Q_d - e_h S_d$$
(134)

Since the net emissions from the cogeneration facility are the same in both the CGPE model and the IPPE model, the difference in the total NO_x emission in the system is based on the NO_x emissions due to electricity generation by the electric utility.

$$x_u^{cgpe} = \frac{\beta - m - P_n e_u - 2\gamma\alpha S_d + 2\gamma Q_d}{2(\gamma + n)} = x_u^{ippe} - \frac{\gamma(\alpha S_d - Q_d)}{2(\gamma + n)}$$

Therefore, $x_u^{cgpe} < x_u^{ippe}$ and the NO_x emissions due to electricity generation by the electric utility is less in the CGPE model than in the IPPE model. Hence if the equilibrium solution of the CGPE model is the non-arbitrage binding solution and the equilibrium solution of the IPPE model is the binding solution 2 then the total NO_x emission in the system will always be less in the CGPE model than in the IPPE model.

If the equilibrium solution of the CGPE model is arbitrage-non-binding solution, the total NO_x emissions generated in the system is

$$TE_{cgpe} = e_u \frac{\beta - m - P_n e_u + \gamma Q_d - 2\gamma(\frac{P_s - c_1}{2c_2})}{2(\gamma + n)} + e_{cg} \frac{P_s - c_1}{2c_2} - e_h S_d$$
(135)



In the arbitrage binding solution of the CGPE model, the NO_x emissions from the qualifying facility increases due to increased electricity generation for sale to the host utility as part of the PURPA contract. In addition the net emissions from the qualifying facility will also increase since at equilibrium the qualifying facility is purchasing electricity from the host utility to satisfy the thermal host's electricity demand. Due to increased sale of electricity by the qualifying facility, the host utility will reduce its own generation quantity. Hence the condition under which the total NO_x emissions in the CGPE model will be less than the IPPE model is

$$e_{cg}\left(\frac{P_{s}-c_{1}}{2c_{2}}-\alpha S_{d}\right) < \frac{e_{u}}{2(\gamma+n)}\left(2\gamma\left(\frac{P_{s}-c_{1}}{2c_{2}}\right)-\gamma\alpha S_{d}-2(n+\gamma)Q_{d}\right)$$
(136)

with $\frac{P_s-c_1}{2c_2} - \Delta = \alpha S_d$. Δ is the additional electricity generated by the qualifying facility because it has a PURPA contract and can sell electricity to the host utility. If the host utility and qualifying facility utilize the same fuel and have the same combustion efficiency their NO_x emission rates will also be the same. Under such a condition where the host utility and qualifying facility emit NO_x emissions at the same rate from condition (136) we have the total NO_x emissions in the CGPE model with non-arbitrage non-binding equilibrium solution mostly be greater than the total NO_x emissions in the IPPE model.

Based on the condition (136) we can determine a upper bound for the emission rate of the qualifying facility which will ensure that that total NO_x emissions in the CGPE model with non-arbitrage non-binding equilibrium solution is less than the total NO_x emissions in the IPPE model with binding solution with $P_r^{ipp} < c_1 + 2c_2\alpha S_d$. This upper bound or limit on the emission rate is



$$e_{cg} < \frac{e_u(2\gamma\Delta + \gamma\alpha S_d - 2(n+\gamma)Q_d)}{2\Delta(\gamma+n)}$$
(137)

If the equilibrium solution of the CGPE model is non-arbitrage-non-binding solution, the total NO_x emissions generated in the system is

$$TE_{cgpe} = e_u \frac{\beta - m - P_n e_u + \gamma Q_d - 2\gamma(\frac{P_s - c_1}{2c_2})}{2(\gamma + n)} + e_{cg} \frac{P_s - c_1}{2c_2} - e_u Q_d - e_h S_d$$
(138)

In the non-arbitrage binding solution of the CGPE model, the NO_x emissions from the qualifying facility increases due to increased electricity generation for sale to the host utility as part of the PURPA contract. However due to increased sale of electricity by the qualifying facility, the host utility will reduce its own generation quantity. Hence the condition under which the total NO_x emissions in the CGPE model will be less than the IPPE model is

$$e_u \frac{{}^{2\gamma(\alpha S_d + \Delta) - \gamma Q_d}}{{}^{2(\gamma+n)}} - e_{cg}(\Delta) > e_u \frac{{}^{\gamma \alpha S_d}}{{}^{2(\gamma+n)}}$$
(139)

with $\frac{P_s-c_1}{2c_2} - \Delta = \alpha S_d$. Δ is the additional electricity generated by the qualifying facility because it has a PURPA contract and can sell electricity to the host utility. If the host utility and qualifying facility utilize the same fuel and have the same combustion efficiency their NO_x emission rates will also be the same. Under such a condition where the host utility and qualifying facility emit NO_x emissions at the same rate from condition (139) we have the total NO_x emissions in the CGPE model with non-arbitrage non-binding equilibrium solution will always be greater than the total NO_x emissions in the IPPE model.

Based on the condition (139) we can determine a upper bound for the emission rate of the qualifying facility which will ensure that total NO_x emissions in the CGPE model with non-arbitrage non-binding equilibrium solution is less than the total NO_x emissions in the



IPPE model with binding solution with $P_r^{ipp} < c_1 + 2c_2\alpha S_d$. This upper bound or limit on the emission rate is

$$e_{cg} < \frac{\gamma e_u(\alpha S_d - Q_d + 2\Delta)}{2\Delta(\gamma + n)}$$
(140)

A summary of the environmental performance comparison between the CGPE and IPPE model when the equilibirum solution of the IPPE model is the binding solution with $P_r^{ipp} < c_1 + 2c_2\alpha S_d$ is shown in Table 10

CGPE equilibrium solution	Total NO _x comparison with HPE model if emission rate of qualifying facility and host utility are the same	Conditions on emission rate of the qualifying/cogeneration facility (e_{cg}) for lower emission in the CGPE model
Arbitrage/Binding	Lower if $\alpha S_d > \frac{2(\gamma+n)}{\gamma} Q_d$	N/A
Arbitrage/Non-binding	Higher	$e_{cg} < \frac{e_u(2\gamma\Delta + \gamma\alpha S_d - 2(n+\gamma)Q_d)}{2\Delta(\gamma+n)}$
Non-arbitrage/Binding	Lower	N/A
Non-arbitrage/Non-binding	Higher	$e_{cg} < \frac{\gamma e_u (\alpha S_d - Q_d + 2\Delta)}{2\Delta(\gamma + n)}$

Table 10: Comparison of environmental performance between the CGPE model and

IPPE model when the equilibrium solution in the IPPE model is binding.

Case (b) Non-Binding solution with $P_r^{ipp} > c_1 + 2c_2\alpha S_d$

The total NO_x emissions in the IPPE model if the equilibrium solution is non-binding with

$$P_r^{ipp} > c_1 + 2c_2\alpha S_d$$
 is

$$TE_{ippe} = e_u \frac{2c_2(\beta + \gamma Q_d - m - P_n e_u) + \gamma(c_1 - m - P_n e_u))}{2(n\gamma + 2(n+\gamma)c_2)} + e_{cg} \frac{P_r - c_1}{2c_2} - e_u Q_d - e_h S_d$$
(141)

Based on the conditions for the equilibrium solution of the IPPE model it can be seen that the parameter values that result in non-binding solution for the IPPE will result only in non-



arbitrage equilibrium solutions in the CGPE model. Hence we only compare the environmental performance of the non-arbitrage mode equilibrium solutions of the CGPE are compared with the non-binding solution of the IPPE model. If the equilibrium solution to the CGPE model is non-arbitrage non-binding then the total emissions from the CGPE model will be less than the total emissions from the IPPE model with non-binding equilibrium solution if the following condition is satisfied

$$e_{cg} < e_u \frac{\gamma(c_1(n\gamma + (n+\gamma)c_2) - n\gamma P_s + 2(n+\gamma)c_2^2 Q_d + c_2(n\beta + m\gamma + \gamma e_u P_n - 2(n+\gamma)P_s + 2n\gamma Q_d))}{2(n+\gamma)c_2(n\gamma + 2(n+\gamma)c_2)(P_r - P_s)}$$
(142)

If the equilibrium solution is non-arbitrage binding solution, then the net emissions from the qualifying facility will be less in the CGPE model than in the IPPE model. This is because since at equilibrium $P_r^{ipp} > c_1 + 2c_2\alpha S_d$, the cogenerated electricity is greater in the IPPE model. However the emissions from the electric utility will increase in the CGPE model since it generates more electricity than in the IPPE model. Hence the total NO_x emissions will be less if the following condition is satisfied

$$e_u \frac{(\gamma(n+\gamma)c_1 - \gamma(n\beta + m\gamma + \gamma e_u P_n + 2(n\gamma + (n+\gamma)c_2)Q_d - 2\alpha(n\gamma + 2(n+\gamma)c_2)S_d)}{2(n\gamma + 2(n+\gamma)c_2)} < e_{cg} \left(\frac{P_r - c_1}{2c_2} - \alpha S_d\right) (143)$$

3.8.2 Comparison of HPE model and CGPE model

The total No_x emissions in the system for the optimal solution in the HPE model is

$$TE_{hpe} = e_u \frac{\beta - m - P_n e_u + \gamma Q_d}{2(\gamma + n)} + e_h S_d$$
(144)

The total No_x emissions associated with the binding equilibrium solutions in the CGPE model is

$$TE_{cgpe} = e_u \frac{\beta - m - P_n e_u - 2\gamma \alpha S_d + \gamma Q_d}{2(\gamma + n)} + e_{cg} \alpha S_d - e_u Q_d - e_h S_d \quad \text{(Arbitrage mode) or}$$



$$TE_{cgpe} = e_u \frac{\beta - m - P_n e_u - 2\gamma \alpha S_d + 2\gamma Q_d}{2(\gamma + n)} + e_{cg} \alpha S_d - e_u Q_d - e_h S_d$$
 (Non-arbitrage mode) (145)
From (144) and (145) we have that the total electricity generation from the electric utility is
lower in the CGPE model. Hence if the net emissions from the qualifying facility are less
than the emissions from the heat production unit then $TE_{cgpe} < TE_{hpe}$.
If the equilibrium solutions of the CGPE model is binding (both arbitrage and non-arbitrage)

then the condition on the emission rate of the qualifying facility that ensures that the total NO_x emissions is lower is

$$e_{cg}\alpha < 2e_h \tag{146}$$

If the equilibrium solution of the CGPE model is the non -binding solutions, the associated total No_x emissions is

$$TE_{cgpe} = e_u \frac{\beta - m - P_n e_u - 2\gamma(\frac{P_s - c_1}{2c_2}) + \gamma Q_d}{2(\gamma + n)} + e_{cg} \frac{P_s - c_1}{2c_2} - e_u Q_d - e_h S_d \quad \text{(Arbitrage mode) or}$$

$$TE_{cgpe} = e_u \frac{\sum_{n=1}^{p} \sum_{n=1}^{m} \sum_{n=1}^{p} \sum_{n=1}^{m} e_{u} Q_d - e_h S_d \text{ (Non-arbitrage mode) (76)}}{2(y+n)} + e_{cg} \frac{\sum_{n=1}^{p} e_{n} Q_d - e_h S_d \text{ (Non-arbitrage mode) (76)}}{2(y+n)}$$

Once again from (144) and (146) we have that the total electricity generation from the electric utility is still lower in the CGPE model than in the HPE model. Hence if the net emissions from the qualifying facility are less than the emissions from the heat production unit then $TE_{cgpe} < TE_{hpe}$. The condition on the emission rate of the qualifying facility that ensures that the total NO_x emissions is lower in the CGPE model is

$$e_{cg} < \frac{2e_h S_d}{\alpha S_d + \Delta} \tag{147}$$



3.8.3 Comparison of SCGE model and CGPE model

The total No_x emissions in the system for the optimal solution in the SCGE model is

$$TE_{scge} = e_u \frac{\beta - m - P_n e_u}{2(\gamma + n)} + e_{cg} \alpha S_d - e_u Q_d - e_h S_d$$
(148)

The total No_x emissions associated with the binding equilibrium solutions in the CGPE model is

$$TE_{cgpe} = e_u \frac{\beta - m - P_n e_u - 2\gamma \alpha S_d + \gamma Q_d}{2(\gamma + n)} + e_{cg} \alpha S_d - e_u Q_d - e_h S_d \quad \text{(Arbitrage mode) or}$$

$$TE_{cgpe} = e_u \frac{\beta - m - P_n e_u - 2\gamma \alpha S_d + 2\gamma Q_d}{2(\gamma + n)} + e_{cg} \alpha S_d - e_u Q_d - e_h S_d \quad \text{(Non-arbitrage mode) (149)}$$
From equations (148) and (149) we see that $TE_{cgpe} < TE_{scge}$. Therefore the CGPE model has greater environmental performance than that of the SCGE model if the equilibrium solution of the CGPE model is binding in the heat constraint of the qualifying facility. If the equilibrium solution of If the CGPE model is the non-binding solution, the associated total No_x emissions is

$$TE_{cgpe} = e_u \frac{\beta - m - P_n e_u - 2\gamma(\frac{P_s - c_1}{2c_2}) + \gamma Q_d}{2(\gamma + n)} + e_{cg} \frac{P_s - c_1}{2c_2} - e_u Q_d - e_h S_d \text{ (Arbitrage mode) or}$$

$$TE_{cgpe} = e_u \frac{\beta - m - P_n e_u - 2\gamma(\frac{P_s - c_1}{2c_2}) + 2\gamma Q_d}{2(\gamma + n)} + e_{cg} \frac{P_s - c_1}{2c_2} - e_u Q_d - e_h S_d \text{ (Non-arbitrage mode) (150)}$$
The condition under which the total emissions from the CGPE model will be less than the

The condition under which the total emissions from the CGPE model will be less than the SCGE model is

$$e_{cg} < e_u \frac{\gamma(\alpha S_d + \Delta - Q_d)}{2\Delta(\gamma + n)} \tag{151}$$



CHAPTER 4. CONCLUSIONS AND FUTURE RESEARCH 4.1 Conclusions

In this thesis we have developed increasing complex models of various configurations of a energy system that consisted of different electric utility- cogeneration facility relationships. We formulated mathematical models and determined the optimal generation plan for the electric utility and the cogeneration facility under the varying scenarios. We compared the total surplus and total No_x emissions of the cogeneration under PURPA configuration with three benchmarks. Specifically, we compared it with the scenario where the cogeneration facility sells electricity directly to retail electricity customers to determine the relative performance of cogeneration under PURPA with a configuration that is most likely in terms of structure.

We found that if in the cogeneration under PURPA configuration the total surplus realized is less than the total surplus realized when the cogeneration facility acts as an Independent Power Producer. Contrary to the claims of the utilities we found that it was the electric utilities that benefited the most under such a situation. The electric utilities received higher profits in the above described scenario since they were able to purchase electricity from the qualifying facility and sell it to the retail electricity customers at a higher rate.

If the qualifying facility only generated the quantity of electricity required to cogenerate the heat demand of the thermal host but sold all the cogenerated electricity to the host utility and purchased from the host utility the electricity required to satisfy the thermal host's electricity demand, then the total surplus level in the cogeneration under PURPA configuration is lower the total surplus level in the configuration where cogeneration facility



as an Independent Power Producer configuration. This is due to the qualifying facility engaging in arbitrage by selling electricity to the host utility at a higher price and purchasing from the host utility at a lower price.

One of the main intents of PURPA was to encourage the cogeneration facilities to sell their surplus electricity which in the absence of PURPA would have been wasted. Due to our comparison of the cogeneration under PURPA configuration with the self-generation with cogeneration configuration we were able to analytically show that sale of the surplus electricity does increase total surplus. By surplus electricity we mean the electricity that is left after the satisfaction of the thermal host's electricity demand from the quantity of electricity generated to cogenerate the heat demand of the thermal host. This validates one of the success of PURPA with reference to one of its intents which was to induce cogeneration facilities to sell their surplus electricity. Also in the scenario where the cogeneration option is not available to the thermal host and it utilized a heat production unit to satisfy its heat demand and purchased electricity from the electric utility, the total surplus was lower than in the cogeneration under PURPA configuration.

In terms of total No_x emissions the cogeneration under PURPA proved to have lower total No_x emissions in the system than the IPP configuration when in both configurations the cogeneration facility generated the quantity of electricity required to cogenerate the heat demand of the thermal host. This was due to the electric utility reducing its generation quantity to maintain electricity prices close to the monopoly price and also to reduce its cost of compliance with the No_x regulatory program. In comparison with the other two benchmarks we found that the cogeneration under PURPA configuration had greater NO_x



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emissions when compared to the configuration where the thermal host is self-generation with cogeneration without interacting with the retail customers or the electric utility. However, the cogeneration under PURPA resulted in a higher amount of No_x emissions if due to high PURPA buyback price the qualifying facility generated more electricity. We also identified conditions on the emission rate of the cogeneration facility under which the total No_x emissions in the cogeneration under PURPA configuration will be less than the total No_x emissions in the benchmark models.

4.2 Discussion

The models analyzed in Chapter 2 and 3 provide us with insights into the limitations and advantages of cogeneration under PURPA. It helped us identify conditions on the PURPA buyback price, the electricity demand of the thermal host and the heat demand of the thermal host based on which the justification for PURPA varied. The main limitations of the study are discussed below.

The electricity and heat output's are fixed at a rigid ratio. This proves helpful in simplifying the initial qualifying facility model but it over estimates the cost of cogeneration. This limitation can be overcome by removing the constraint and replacing it with a fuel cost based production function as in Fox-Penner (1990).

The thermal demand is satisfied by cogeneration alone. This again limits the flexibility of the model and creates a situation where the qualifying facility might behave sub optimally. This limitation can be overcome by including a ancillary boiler in the model.


The environmental performance of a configuration can be measured in terms of two indicators – the total No_x emissions in the system and the total usable energy generated in the system.Estimating only the No_x emissions will not completely estimate the environmental performance of a configuration, since a system configuration might have lower emissions due to lower energy generation. The best environmental performance of a energy generation system configuration is one which results in high useful energy generation with low emissions. The second preferable configuration in terms of environmental performance is the system than results in lower emissions and lower useful energy. The least preferable configuration is the one that results in high emissions but low useful energy. An emphasis is placed on useful or usable energy products in the system. If generated useful heat or electricity is not utilized then it should not be considered into the calculations and should be treated as waste.

4.3 Future Research

The problem studied in this paper can be futher explored by considering the effect of transmission constraints and capacity constraints. Specifically, transmission constraints are a critical to understand the arbitrage that the qualifying facility engages in in the cogeneration under PURPA configuration. Will the qualifying facility still sell all its cogenerated electricity to the host utility and purchase the electricity demand for the thermal host if transmission costs and constraints are considered is a question that should be explored. Also we do not consider the scenario where the thermal host might have more electricity demand than heat demand. Even though most of the qualifying cogeneration facilities that



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exist in the United States are of the type to have higher heat demand than electricity demand, the vice-versa situation might high light more insights into PURPA's workings. However, it should be noted that Joskow and Jones (1983) claim that cogeneration facilities that are dedicated to serve the heat load have more economic benefits due to PURPA than the facilities' with more electricity demand. Other extensions might include introducing stochastic elements into the generation planning models



APPENDIX A. CONCAVITY OF PROFIT FUNCTIONS

Proof of concavity of π_{qf}^{cgp} w.r.t x_{cg}

$$\frac{\partial \pi_{qf}^{cgp}}{\partial x_{cg}} = P_s - b - 2cx_{cg} - \frac{d}{\alpha} - 2\frac{e}{\alpha^2}x_{cg} - 2\frac{f}{\alpha}x_{cg}$$
(B.1)
$$\frac{\partial^2 \pi_{hu}^{cgp}}{\partial x_{cg}^2} = -2(c + \frac{f}{\alpha} + \frac{e}{\alpha^2})$$
(B.2)

Since, c, f, e and α are all positive parameters, we have from equation (C.2), that the second

order derivative, $\frac{\partial^2 \pi_{qf}^{cgp}}{\partial x_{cg^2}} < 0$, \forall feasible x_{cg} . Hence, π_{qf}^{cgp} is concave w.r.t x_{cg} .

Proof of concavity of π_{hu}^{cgp} w.r.t x_u in Non-Arbitrage Mode

$$\frac{\partial \pi_{hu}^{cgp}}{\partial x_u} = \beta - 2\gamma \left(x_u + x_{cg} - Q_d \right) - m - 2nx_u$$

$$\frac{\partial^2 \pi_{hu}^{P_6}}{\partial x_u^2} = -2\gamma - 2n = -2(\gamma + n)$$
(B.3)
(B.4)

Since, $\gamma \& n$, are both positive, we have from equation (B.4), that the second order derivative

$$\frac{\partial^2 \pi_{qf}^{P_6}}{\partial x_u^2}$$
 < 0, ∀ feasible x_u . Hence, π_{hu}^{cgp} is concave w.r.t x_u .

Proof of concavity of π_{hu}^{P7} w.r.t x_u in Arbitrage Mode

$$\frac{\partial \pi_{hu}^{cgp}}{\partial x_u} = \beta - 2\gamma \left(x_u + x_{cg} - Q_d \right) - m - 2nx_u - \gamma Q_d \tag{B.5}$$

$$\frac{\partial^2 \pi_{hu}^{cgp}}{\partial x_u^2} = -2\gamma - 2n = -2(\gamma + n)$$
(B.6)

Since, $\gamma \& n$, are both positive, we have from equation (B.6), that the second order derivative

$$\frac{\partial^2 \pi_{hu}^{cgp}}{\partial x_u^2} < 0, \forall$$
 feasible x_u . Hence, π_{hu}^{P7} is concave w.r.t x_u .



APPENDIX B. DETAILS OF COMPARISON OF TOTAL SURPLUS BETWEEN CGP CONGIGURATION AND IPP CONFIGURATION

I. Proof that $TS_{cgp} < TS_{ipp}$ if the equilibrium solution of the CGP model is the arbitrage mode binding solution and the equilibrium solution of IPP is the binding solution.

$$\begin{aligned} x_u^{ipp-B-2} &= \frac{\beta - m - \gamma (\alpha S_d - Q_d)}{2(n+\gamma)}; x_{cg}^{ipp}(P_r) = \alpha S_d; Q_T^{ipp} = \frac{\beta - m - \gamma (\alpha S_d - Q_d)}{2(n+\gamma)} + \alpha S_d - Q_d \\ x_u^{cgp-AM-B} &= \frac{\beta - m - 2\gamma \alpha S_d + \gamma Q_d}{2(n+\gamma)}; x_{cg}^{cgp} = \alpha S_d; Q_T^{cgp} = \frac{\beta - m - 2\gamma \alpha S_d + \gamma Q_d}{2(n+\gamma)} + \alpha S_d - Q_d \end{aligned}$$

Condition for total surplus in the arbitrage mode binding solution of the CGP model to have higher total surplus than the IPP-Binding 2

$$\begin{split} \beta Q_{T}^{ipp} &- \frac{\gamma}{2} Q_{T}^{ipp^{2}} - \beta Q_{T}^{cgp} + \frac{\gamma}{2} Q_{T}^{cgp^{2}} < l + m x_{u}^{ipp} + n x_{u}^{ipp^{2}} - l - m x_{u}^{cgp} - n x_{u}^{cgp^{2}} \\ \text{We know, } Q_{T}^{ipp} - Q_{T}^{cgp} &= \frac{\gamma \alpha S_{d}}{2(\gamma + n)} = x_{u}^{ipp} - x_{u}^{cgp} \\ \beta \frac{\gamma \alpha S_{d}}{2(\gamma + n)} - \frac{\gamma}{2} (Q_{T}^{ipp^{2}} - Q_{T}^{cgp^{2}}) < m \frac{\gamma \alpha S_{d}}{2(\gamma + n)} + n (x_{u}^{ipp^{2}} - x_{u}^{cgp^{2}}) \\ \beta \frac{\gamma \alpha S_{d}}{2(\gamma + n)} - \frac{\gamma}{2} (Q_{T}^{ipp^{2}} - Q_{T}^{cgp^{2}}) - m \frac{\gamma \alpha S_{d}}{2(\gamma + n)} - n (x_{u}^{ipp^{2}} - x_{u}^{cgp^{2}}) < 0 \end{split}$$
(2)
Simplifying using Mathematica, we get $\frac{\gamma^{2} \alpha S_{d} (2\beta - 2m + 2\gamma Q_{d} + (2n - \gamma)\alpha S_{d})}{8(n + \gamma)^{2}} < 0$

→ 2β - 2m + 2γQ_d + (2n - γ)αS_d < 0 →β - m + γQ_d < ($\frac{\gamma}{2}$ - n)αS_d

Since $x_u^{ipp} > 0$, we have $\beta + \gamma Q_d - m > \gamma \alpha S_d$. Hence the above condition will not be satisfied.



II Proof that $TS_{cgp} < TS_{ipp}$ if the equilibrium solution of the CGP model is the nonarbitrage mode binding solution and the equilibrium solution of IPP is the binding solution.

$$\begin{aligned} x_u^{ipp-B-2} &= \frac{\beta - m - \gamma(\alpha S_d - Q_d)}{2(n+\gamma)}; x_{cg}^{ipp}(P_r) = \alpha S_d; Q_T^{ipp} = \frac{\beta - m - \gamma(\alpha S_d - Q_d)}{2(n+\gamma)} + \alpha S_d - Q_d \\ x_u^{cgp-AM-B} &= \frac{\beta - m - 2\gamma\alpha S_d + 2\gamma Q_d}{2(n+\gamma)}; x_{cg}^{cgp} = \alpha S_d; Q_T^{cgp} = \frac{\beta - m - 2\gamma\alpha S_d + 2\gamma Q_d}{2(n+\gamma)} + \alpha S_d - Q_d \end{aligned}$$

Condition for total surplus in the non-arbitrage mode binding solution of the CGP model to have higher total surplus than the IPP-binding

$$\begin{split} \beta Q_T^{ipp} &- \frac{\gamma}{2} Q_T^{ipp^2} - \beta Q_T^{cgp} + \frac{\gamma}{2} Q_T^{cgp^2} < l + m x_u^{ipp} + n x_u^{ipp^2} - l - m x_u^{cgp} - n x_u^{cgp^2} \\ \Rightarrow \beta \left(Q_T^{ipp} - Q_T^{cgp} \right) - \frac{\gamma}{2} (Q_T^{ipp^2} - Q_T^{cgp^2}) < m (x_u^{ipp} - x_u^{cgp}) + n (x_u^{cgs^2} - x_u^{cgp^2}) \\ We \text{ know, } Q_T^{ipp} - Q_T^{cgp} = \frac{\gamma (\alpha S_d - Q_d)}{2(\gamma + n)} = x_u^{ipp} - x_u^{cgp}. \quad \text{Therefore (1) becomes} \\ \beta \frac{\gamma (\alpha S_d - Q_d)}{2(\gamma + n)} - \frac{\gamma}{2} (Q_T^{ipp^2} - Q_T^{cgp^2}) < m \frac{\gamma (\alpha S_d - Q_d)}{2(\gamma + n)} + n (x_u^{ipp^2} - x_u^{cgp^2}) \end{split}$$

$$\beta \frac{\gamma(\alpha S_d - Q_d)}{2(\gamma + n)} - \frac{\gamma}{2} (Q_T^{ipp^2} - Q_T^{cgp^2}) - m \frac{\gamma(\alpha S_d - Q_d)}{2(\gamma + n)} - n(x_u^{ipp^2} - x_u^{cgp^2}) < 0$$
(2)

Simplifying using Mathematica, we get

$$\frac{\gamma^{2}(Q_{d}-\alpha S_{d})(2(m-\beta)+(2n-\gamma)(Q_{d}-\alpha S_{d}))}{8(n+\gamma)^{2}} < 0$$

$$2(m-\beta) + (2n-\gamma)(Q_{d}-\alpha S_{d}) < 0$$

$$2m-2\beta - (2n-\gamma)(\alpha S_{d}-Q_{d}) < 0$$

$$2m+\gamma(\alpha S_{d}-Q_{d}) < 2\beta + 2n(\alpha S_{d}-Q_{d})$$

$$\left(\frac{\gamma}{2}-n\right)(\alpha S_{d}-Q_{d}) < \beta - m$$
integers

Since $x_u^{ipp} > 0$, we have $\beta - m > \gamma(\alpha S_d - Q_d)$.



BIBLIOGRAPHY

Adelberg A. (1999) "Way off on PURPA". Letters to the Editor, response to Hirsh, R.F's article "PURPA: The Spur to Competition and Utility Restructuring". The Electricity Journal;

Association of Electric Companies of Texas, Inc (ACET). (2007) "Vertically Integrated Markets in Texas". Retrieved on 11/15/08 from

http://www.aect.net/documents/2007/20070102_BK_VertInt.pdf

Cachon G and Netessine S (2003) 'Game theory in Supply Chain Analysis. Supply Chain Analysis in the e-Business Era'', edited by Simchi-levi et al. Kluwer;

Chapa G and Galaz V. (2004) "An economic dispatch algorithm for cogeneration systems". Power Engineering Society General Meeting ; 1; 989-994

Chen BK and Hong CC.(1996) "Optimum operation of back pressure cogeneration system under time of use rates". IEEE Transactions on Power Systems; 11; n2; 1074-1082.

Chen Y and Hobbs BF. (2005) "An Oligopolistic Power Market Model with tradable

No_x permits". IEEE Transactions on Power Systems; 20; 1; 119-129.

Chen Y, Hobbs BF, Leyffer S and Munson T. (2006) "Leader-Follower Equilibira for Electric Power and No_x Allowance Markets". Computational Management Science; 3; 4; 307-330.

Danielsen AL, Gupta NK and Klein PG. (1999) 'Contracts and the Institutional Environment for Electricity Reform'. The Electricity Journal; 12; 10; 51-60.

Energy Information Administration, 2000. The Changing Structure of the Electric Power Industry 2000: A Update. Chapter 6, Section: Repeal of PURPA. Retrieved on



12/15/07 from

http://www.eia.doe.gov/cneaf/electricity/chg_stru_update/chapter6.html#N_12_

Energy Information Administration 2006a. Form EIA-906 and EIA-920 Databases.

Retreived on 11/14/08 from

http://www.eia.doe.gov/cneaf/electricity/page/eia906_920.html

Energy Information Administration, 2008. Status of Electricity Restructuring by

State. Retrieved on 10/17/08 from

http://www.eia.doe.gov/cneaf/electricity/page/restructuring/restructure_elect.html

Environmental Protection Agency (EPA). Acid Rain Program. Retrieved on 1/4/08

from http://www.epa.gov/airmarkets/progsregs/arp/basic.html

Fox-Penner P. (1990) "Cogeneration after PURPA: Energy Conservation and

Industry Structure". Journal of Law and Economics; 33; 517-552.

Galssman LW (2001). "Qualifying Facilities in the Post EPAct World: Challenges

and Opportunities for QF's Part II of III". Electric Light & Power Magazine; March-April

Issue. Retrieved on 12/01/07 from

http://uaelp.pennnet.com/display_article/289078/34/ARTCL/none/none/1/Qualifying--

Facilities-in-the-Post-EPAct-World/

Gibbons, R. (1992) "Game theory for Applied Economists". Princeton University Press: Princeton, NJ;.

Gottlieb JW.(2001) "How would PURPA Repeal affect Cogeneration?" Cogeneration and Competitive Power Journal; 16; 1; 47-48.



Guo T, Henwood MI and Van Ooijen M. (1996) "An algorithm for combined heat and power economic dispatch". IEEE Transactions on Power Systems; 11; 4; 1778-1784.

Gupta BR. (1983.) "Generation of Electricity", 1st Edition. Eurasia Publishing House: New Delhi;

Haurie A, Loulou R and Savard G. (1992) "A two-player game model of power cogeneration in New England". IEEE Transactions on Automatic Control; 37; 9; 1451-1456.

Hawley S on behalf of the Watson Cogeneration Company. In the Matter of: 2005 Integrated Energy Policy Report CHP workshop April 28, 2005. Docket 04-IEP-1E.

Hirsh RF. (1999) "PURPA: The Spur to Competition and Utility Restructuring". The Electricity Journal; 12; 7; 60-72.

Hu D. (1985) "Cogeneration". Reston Publishing Company Inc: Reston, VA.

Joskow P and Jones DR. (1983) "The simple economics of industrial cogeneration." The Energy Journal; 4; 1; 1-22.

Kwun YH and Baughman ML. (1991) "Joint Optimal Planning of Industrial Cogeneration and Conventional Electricity Systems." Operations Research; 39; 5; 798-806.

Lamoureux M. (2002) "U.S Electric Energy Policy." IEEE Power Engineering Review; 22; 7; 12-17.

Moleshi K, Khadem M, Bernal R and Hernandez G. (1991) "Optimization of multiple cogeneration system operation including electric and steam networks." IEEE Transactions of Power Systems; 6; 481-490.

Phillipson L and Willis HL (2006). "Understanding Electric Utilities and De-Regulation". Second Edition. Taylor and Francis, New York.



Puttgen HB and MacGregor P. (1989) "Optimum scheduling procedure for cogenerating small power producing facilities". IEEE Transactions on Power systems; 4; 3; 957-964.

Rao N.(2001) "Combined Heat and Power Economic Dispatch: A Direct Approach". Electric Power Components and System; 34; 9; 1043-1056.

Rose A, Peterson TD and Zhang Z. "Regional Carbon dioxide Permit Trading in United States: Coalition Choices for Pennsylvania". Penn State Law Review 2006. Retrieved on 1/6/08 from <u>http://www.geog.psu.edu/pdf/RPZ_final_format_4-7-06.pdf</u>

Rose K and McDonald JF. (1991) "Economics of electricity self-generation". The Energy Journal; 12; 2; 47-66.

Rochester Public Utilities (RPU). (2006) . RPU Facilities. Retrieved on 11/18/2008 from http://www.rpu.org/about/facilities/power/

Rong A and Lahdelma R (2007) "CO2 emissions trading planning in combined heat and power production via multi-period stochastic optimization". European Journal of Operational Research; 176; 1874-1895.

Spiewak S and Weiss L.(1997) "Cogeneration and Small Power Production Manual." 5th Edition. Fairmont Press Inc: Liburn, GA;.

Sundberg G and Henning D (2002). "Investments in combined heat and power plants: influence of fuel price on cost minimized operation". Energy Conversion and Management; 43; 5; 639-650.



Tsay MT, Lin WM and Lee JL (2001) "Application of evolutionary programming for economic dispatch of cogeneration systems under emission constraints". International Journal of Electrical power and Energy systems; 23; 8; 805-812.

Venkatesh BN and Chankong V.(1995) "Decisions Models for Management of Cogeneration plants". IEEE Transactions on Power Systems; 10; 3; 1250 – 1256.

Venkatesh P, Gnanadass, R and Padhy, P (2003). "Comparison and application of evolutionary programming techniques to combined economic emission dispatch with line flow constraints". IEEE Transactions on Power Systems; 18; 2; 688-697.

Wilson N, Palmer KL and Burtaw D. (2005) "The Impact of Long term Generation Contracts on Valuation of Electricity Generating Assets under the Regional Greenhouse Gas Initiative". Resources for the future –Discussion Paper. Retrieved on 11/30/07 from http://www.rff.org/documents/RFF-DP-05-37.pdf

Woo CK (1988). "Inefficiency of avoided cost pricing of cogenerated power". The Energy Journal; 9; 1; 103-113.

Wu YJ and Rose MA (1999). "Assessing and optimizing the economic and environmental impacts of cogeneration/district energy systems using an energy equilibrium model". Applied Energy; 62; 3; 141-154.

